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РАЗБИВКА ПЕРЕДАТОЧНОГО ЧИСЛА ЧЕРВЯЧНО-КОНИЧЕСКОГО РЕДУКТОРА ПО СТУПЕНЯМ

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Ключевые слова: редуктор, ступень, передаточное число редуктора, червячная передача, коническая передача, условие разбивки передаточного числа.

Аннотация. Проанализированы варианты разбивки передаточного числа червячно-конического редуктора по ступеням. Получены расчетные зависимости для определения передаточных чисел ступеней редуктора из условия смазки обеих ступеней. Рассмотрены два варианта конической передачи – с прямыми и круговыми зубьями. Разбивка передаточного числа червячно-конического редуктора из условия минимизации периметра опорной поверхности редуктора обеспечивает компактность занимаемой редуктором площади. Расчеты выполнены для различных сочетаний материалов червячного колеса и способов упрочнения зубьев конической передачи.

Formulation of the problem

The partitioning of the gear ratio of a multi-stage gearbox by steps is a multivariate task [1-3]. The results of its solution largely determine the technical characteristics of the gearbox, in particular, the lubrication conditions of its stages, overall dimensions, uniform strength, etc [4, 5].

The gear ratio $U = U_I \cdot U_{II}$ of mass-produced designs of two-stage gearboxes with input and output shafts parallel axes does not exceed (50 ... 60). This is due to a constructive restriction on gear ratios $U_I = 8$ and $U_{II} = 6{,}3$ two cylindrical gears, respectively, of the high-speed and low-speed stages of these gears, [1]. One of the ways to significantly increase the values U of gears with parallel axes of the input and output shafts was proposed in [6] – the use of a worm-bevel gearbox (WB), figure 1.

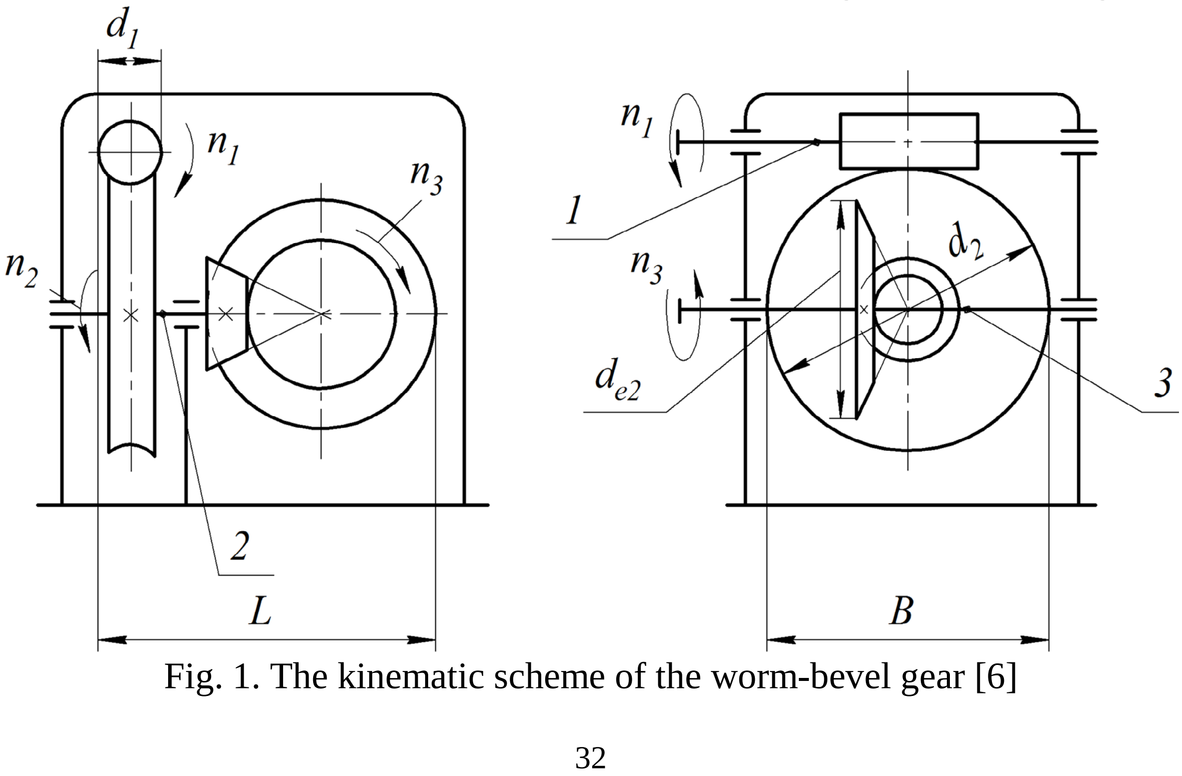

Fig. 1. The kinematic scheme of the worm-bevel gear [6]

The use of WB gearboxes will allow, under the current restrictions on worm $(U_I \le 80)$ and bevel $(U_{II} \le 6)$ gear ratios to significantly increase U the gearbox – up to $U \approx 150...200$ and more [1]. This, in turn, will make it possible by $U > 80$ to replace cumbrous 3-stage spur gearboxes with more compact 2- stage WB gearboxes. The question boils down to a rational partitioning of the value U of the WB gearbox between the worm gear (U_I) and the bevel (U_{II}) gear. The basis of this partitioning can be based on various conditions: the minimum of the WB base surface $(S_{\min})$, the minimum of the BW volume $(V_{\min})$, the equal strength of the stages on the contact endurance of the teeth, the condition of lubrication of both stages, etc. This article presents the results of a study on the partitioning of the WB gear ratio gearbox both steps from the condition of minimizing the perimeter of the base surface of the BW gearbox.

1. The partitioning of the WB gearbox from the condition of lubrication

Fulfillment of the condition of the partitioning by the BW steps with lubrication will provide the same depth of immersion of the wheels for both stages of the gearbox into the oil bath. This condition is reduced to equality:

$$d_2 = d_{e2}, \tag{1}$$

where d_2 и d_{e2} – the pitch diameter of the worm wheel and the outer pitch diameter of the bevel wheel, respectively.

Parameters d_2 и d_{e2} we will find, using the main criteria of the worm and bevel gear performance – the contact endurance of the teeth, with the maximum level of their loading, that is, with $\sigma_{H(I)} = [\sigma_H]_I$ и $\sigma_{H(II)} = [\sigma_H]_{II}$ (index I refers to worm transmision, index II – to bevel transmission), [7]:

$$\sigma_{H(I)} = \frac{5300}{z_2 / q} \cdot \sqrt{\left(\frac{1 + z_2 / q}{a_W}\right)^3 \cdot K_{H(I)} \cdot T_2} = [\sigma_H]_I; \tag{2}$$

$$\sigma_{H(II)} = 2120 \cdot \sqrt{\frac{1000 \cdot K_{H(II)} \cdot T_3 \cdot U_{II}}{\theta_H \cdot d_{e2}^3}} = [\sigma_H]_{II}. \tag{3}$$

For equality (2)**:**

$$q = z_2 / 4; \quad a_W = C_I \cdot U^{-1/3} \cdot U_I^{1/3}; \quad K_{H(I)} \approx 1{,}1; \quad T_2 \approx T_3 / U_{II}, \tag{4}$$

where $C_I = 5 \cdot \sqrt[3]{\left(\frac{5300}{4 \cdot [\sigma_H]_I}\right)^2 \cdot K_{H(I)}} = const$. Wherein:

$$d_2 = m \cdot z_2 = \frac{2 \cdot a_W}{z_2 + q} \cdot q = \left\{ \begin{matrix} Для \\ q = z_2 / 4 \end{matrix} \right\} = 1{,}6 \cdot a_W = 1{,}6 \cdot C_I \cdot U^{-1/3} \cdot U_I^{1/3}, \tag{5}$$

where $a_W = \left(1 + \frac{z_2}{q}\right) \cdot \sqrt[3]{\left(\frac{5300}{[\sigma_H]_I \cdot z_2 / q}\right)^2 \cdot K_{H(I)} \cdot T_2} = C_I \cdot U^{-1/3} \cdot U_I^{1/3}$.

For equality (3):

$$K_{H(II)} \approx 1{,}3; \quad d_{e2} = C_{II} \cdot U_{II}^{1/3} \cdot \theta_H^{-1/3}; \quad \theta_H = c_1 + c_2 \cdot U_{II} = c_1 + c_2 \cdot U \cdot U_I^{-1}, \tag{6}$$

where $C_{II} = 1650 \cdot \sqrt[3]{\frac{K_{H(II)}}{[\sigma_H]_{II}^2}} = const.$; c_1 and c_2 – constants depending on the hardness of the gear teeth and the bevel gear wheel [7].

Tabl. 1.

Hardening gears / wheels	C_I	C_{II}
HR_1/HR_2	1,22	0,21
HIH_1/HR_2	1,13	0,13
HIH_1/HIH_2	0,81	0,15

Hear: HR_1/HR_2 – heat refining pinion (1) and wheel (2) bevel transmission; HIH_1/HR_2 – high induction hardering tooth of pinion and heat refining wheel tooth; HIH_1/HIH_2 – high induction hardering tooth of pinion and wheel.

With these substitutions, the partitioning condition (1) transforms into the following equation:

$$F_1(U_I) = 1{,}6 \cdot C_I \cdot U^{-1/3} \cdot U_I^{1/3} - C_{II} \cdot U^{1/3} \cdot U_I^{-1/3} \cdot \theta_H^{-1/3} = 0. \quad (7)$$

1.1. Solving equation (7) for spur bevel gears concerning U_I, that is, for $\theta_H = 0{,}85$, [7], leads to a linear relationship between U_I and U:

$U_I = k_1 \cdot U$, (8)

where $k_1 = 0{,}54 \cdot \sqrt{(C_{II} / C_I)^3} = const$. The values of the constants k_1 are given in table 2.

Tabl. 2.

№	1	2	3	4	5	6
k_1	0,92	0,78	0,57	–	0,97	0,71
U	≤87	≤102	≤140		≤82	≤112

Numbers № 1 ... № 6 correspond to the following combinations of materials of the teeth of the worm wheel and methods of hardening the gear teeth and the wheel of bevel gear:

1 → TLB+ HR_1/HR_2; 2 → TLB + HIH_1/HR_2; 3 → TLB + HIH_1/HIH_2;

4 → TB+ HR_1/HR_2; 5 → TB+ HIH_1/HR_2; 6 → TB + HIH_1/HIH_2;

where “TB” и “TLB” – tin bronze and tinless bronze at the teeth of the worm wheel.

The bottom line of the table. 2 given limit values U for which the recommendation is made for power worm gears: $U_I \leq 80$. Dashes in the table. 2 means that for combinations of constants from No. 4, a partitioning U of the WB gearbox [8-10] is not possible according to the lubrication condition, since it is a value $U_I > 80$. From table 2 it follows that the greatest opportunities for a partitioning U from the condition of wheels lubrication at the WB gearbox are for No. 3, where the limitation $U_I \leq 80$ is fulfilled when $U \approx 80...112$.

1.2. For a bevel gear [11, 12] with *circular teeth*, that is, with $\theta_H = c_1 + c_2 \cdot U_{II}$, function (7) is converted to a quadratic equation for U_I: $U_I^2 + b_1 \cdot U_I + b_2 = 0$, one of the roots of which is defined by the expression:

$$U_I = -0{,}5 \cdot b_1 + \sqrt{0{,}25 \cdot b_1^2 - b_2}\ , \qquad (9)$$

here $b_1 = -(0{,}244 / c_1) \cdot (C_{II} / C_I)^3 \cdot U^2$; $b_2 = (c_2 / c_1) \cdot U$.

For the convenience of practical use of the proposed method of dividing U the steps of the WB gearbox, the function (9) is approximated by a power dependence of the form $U_I = k_2 \cdot U^t$. Numerical calculations for the operating range $U_I \approx 10...80$ showed that the power dependence is practically transformed into a linear

$$U_I = k_2 \cdot U\ , \qquad (10)$$

as an exponent $t \approx 1$. The numerical values of the coefficient k_2 are given in table 3.

Tabl. 3.

№	1	2	3	4	5	6
k_2	0,67	0,66	0,33	0,68	0,79	0,66
U	≤ 122	≤ 132	≤ 165	≤ 96	≤ 105	≤ 96

According to the found U_I is corresponding to him $U_{II} = U / U_I$. As can be seen from the table 3, the ranges U for different combinations of constants under which the constraint $U_I \le 80$ is maintained differ significantly: from $U \approx 70...96$ for No. 6 to $U \approx 70...165$ for No. 3.

2. The partitioning U of the WB gearbox from the condition of the minimum perimeter of the housing base

The compactness of the area occupied by the WB gearbox is achieved by minimizing the perimeter of its base $P = 2 \cdot (L + B)$ (dimensions L and B in figure 1). Using the above analytical dependencies for the parameters of the worm and bevel gears, we obtain the calculated ratios for the dimensions of the WB gearbox base surface – $L = L(U_I)$ and $B(U_I)$**:**

$$L = d_1 + d_{e2} = 0{,}4 \cdot C_I \cdot U^{-1/3} \cdot U_I^{1/3} + C_{II} \cdot U_{II}^{1/3} \cdot \theta_H^{-1/3};$$

$$B = d_2 = 1{,}6 \cdot C_I \cdot U^{-1/3} \cdot U_I^{1/3}.$$

As a result**:**

$$P = P(U_I) = 2 \cdot (2 \cdot C_I \cdot U^{-1/3} \cdot U_I^{1/3} + C_{II} \cdot U^{1/3} \cdot U_I^{-1/3} \cdot \theta_H^{-1/3})\ . \qquad (11)$$

Presented as an example graphically in figure 2 functions (11) indicate the presence of their values $P = P_{\min}$.

Specific values U_I found from the equation $\partial P(U_I) / \partial U_I = 0$ and corresponding to the condition $P = P_{\min}$ are calculated by the formula:

$$U_I = C \cdot U\ , \qquad (12)$$

where $C = 0{,}38 \cdot \sqrt{(C_{II} / C_I)^3}$ – constant table 4.

Tabl. 4.

№	1	2	3	4	5	6
C	0,65	0,55	0,40	0,80	0,69	0,50

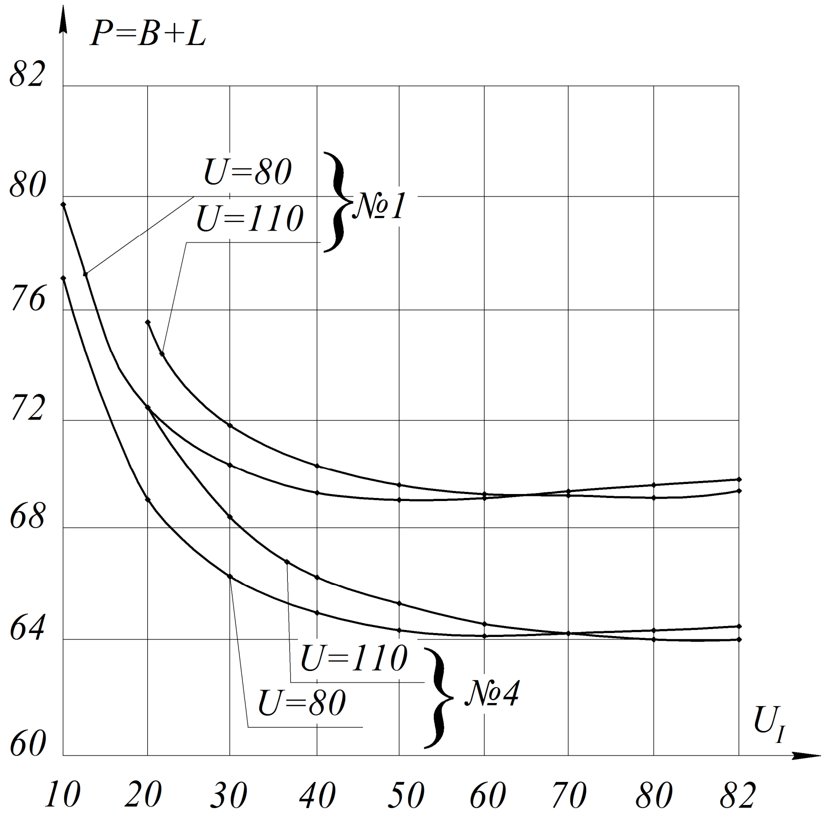

Fig. 2. The function of the half-perimeter of the base WB gearbox

The calculations of the constant C are made for a WB gearbox with a spur bevel gear. In the WB reducer, in which the bevel gear has circular teeth, as shown by the numerical analysis of the equation roots (11), a partitioning U by condition $P = P_{\min}$ is not possible, because the gear ratios of the first stage $U_I > 80$.

Formula (12) and the values of the constant C are valid for the following ranges U of the WB reducer:

№1 – $U \approx 70...125$; №2 – $U \approx 70...145$; №3 – $U \approx 70...200$;
№4 – $U \approx 70...100$; №5 – $U \approx 70...115$; №6 – $U \approx 70...160$.

Outside the specified ranges, the use of WB gearboxes will lead to values U_I or U_{II}, exceeding their maximum recommended values.

Conclusions

A technique for partitioning the gear ratio of a worm-bevel gearbox in steps from the lubrication condition, in which lubrication is provided by immersing the wheels of both gears in oil bath has been developed. The calculated dependencies for the gear ratios U_I and U_{II} WB gearbox stages, which satisfy this condition under the existing restrictions on the values U_I and U_{II}, are obtained. In this case, two variants of a bevel gear are considered – with straight and circular teeth.

A technique for partitioning the gear ratio of the WB gearbox based on the condition of minimizing its base surface perimeter is proposed. This approach in the partitioning U of the WB gearbox if the bevel gear of the gearbox has straight teeth can be implemented.

The use of the WB gearbox in drives with parallel arrangement of the axes of the input and output shafts is an alternative to two-stage helical gearboxes, for which

$U = 6{,}3...50$, and three-stage helical gearboxes, which can be implemented U in the WB gearbox $U = 16...250$, but significantly longer than them.

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RATIO PARTITIONING OF WORM-BEVEL GEARBOX BY STEPS

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Keywords: gearbox, stage, gearbox ratio, worm gear, bevel gear, the condition of the gear ratio partitioning.

Abstract. Options for the partitioning of the gear ratio of the worm-bevel gear in steps are analyzed. The calculated dependences for determining the ratios of the gearbox stages from the condition of lubrication of both stages are obtained. Two variants of bevel gear are considered – with straight and circular teeth. The partitioning of the ratio for the worm-bevel gearbox from the condition of minimizing the perimeter of the gearbox base surface ensures compactness of the area occupied by the gearbox. The calculations for various combinations of worm wheel materials and methods of hardening the teeth of a bevel gear are performed.