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PROPERTIES OF CONSTRUCTED OPTIMAL OBJECT MOTIONS

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Abstract. The characteristic features of the reversive design of optimal controls such as acceleration-deceleration of the translational motion of objects as absolutely rigid bodies from the initial state to the final state of absolute quiescence with the assignment (as initial data) of the time of movement and distance are revealed. The influence of the degree of the specified displacement polynomial on the energy intensity of the optimal movement is estimated. The examples show that when implementing the control design algorithm (in the form of polynomials), there appear functional-criteria that confirm the optimality of control synthesis.

СВОЙСТВА КОНСТРУИРУЕМЫХ ОПТИМАЛЬНЫХ ДВИЖЕНИЙ ОБЪЕКТОВ

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Ключевые слова: разгон-торможение, реверсионное конструирование, типы оптимальных управлений, снижение энергоемкости движения.

Аннотация. Выявлены характерные особенности реверсионного конструирования оптимальных управлений типа разгон-торможение переносным движением объектов как абсолютно твердых тел из исходного в конечное состояние абсолютного покоя с заданием (в качестве исходных данных) времени движения и расстояния. Оценено влияние степени задаваемого полинома перемещения на энергоемкость оптимального движения. На примерах показано, что при реализации алгоритма конструирования управлений (в виде полиномов) появляются функционалы-критерии, подтверждающие оптимальность синтеза управлений.

Introduction

The extremality of the matter’s motion was most successfully and rigorously noted by L. Euler in his work “The Method for Finding Curved Lines Having a Maximum or a Minimum”. L. Euler writes: “Nothing happens in the world in which the meaning of any maximum or minimum would not be visible;... all phenomena of the world can just as well be determined from the finite causes ... so from the producing causes “.

This statement to a certain extent echoes the one formulated in 1829 K. Gauss principle of least coercion [1]. The modern place of the principle in dynamics is quite fully reflected, for example, in work [2].

In the theory of optimal control, the Pontryagin maximum principle (PMP), the discovery of which the article [3] is devoted, is central. The justification of the PMP is also possible from the standpoint of the classical calculus of variations (Lagrange problem).

With the development of reversion calculus [4] in work [5], as a result of generalization of the algorithm for solving the complete inverse problem of variation calculus (from a given analytical function, obtaining Euler's equation and restoring the functional-criterion), the reversion principle of optimality (RPO) is justified and formulated, which is extended to solving the problems of synthesizing control by translational movement of elastic systems.

The recovered reversibly functional-criterion for the minimum possible time of movement, determined from moment relations in relative motion, takes the minimum value.

The use of optimal control design algorithms is illustrated using examples of control of movement of engineering objects [6, 7].

Noteworthy is the remark given in the reference book [8]: "In general, however, it is quite difficult to construct the functional *J(y)* according to a given differential equation." Fairly simple cases of functional recovery when solving complete inverse problems of variational calculus are summarized in table [5].

And until now, due attention has not been paid to the restoration of functionals for the specified functions. In practice, it is possible to appear new types of functionals that were not previously used in the tasks of synthesizing optimal controls.

The purpose of the research is to systematize and analyze the properties of the designed controls for the objects motions.

Control Design Examples

1. At polynomial degree $n = 3$ for the displacement $S(t)$ a polynomial is accepted

$$S(t) = \sum_{j=1}^{6} C_i t^{i-1} \qquad (1)$$

with boundary conditions:

$$S(0) = 0, \quad V(0) = 0; \quad S(T) = L, \quad V(T) = 0, \qquad (2)$$

L – predetermined displacement, T – total motion time.

The skew symmetry conditions are as follows:

$$U\left(\frac{T}{2}\right) = 0; \ \frac{dU}{dt}\left(\frac{T}{2}\right) = 0. \qquad (3)$$

Constants $C_1 \ldots C_6$ are found from a system of linear algebraic equations using the function *solve* (Maple).

After the factorization of polynomials, expressions are obtained for $S(t)$, $V(t)$, $U(t)$. So the displacement

$$S(t) = \frac{Lt^2(5T^3 - 10T^2t + 10t^2T - 4t^3)}{T^5} \quad (4)$$

and accordingly follows:

$$V(t) = \frac{dS(t)}{dt}, \qquad U(t) = \frac{dV(t)}{dt}. \quad (5)$$

Polynomial (1) is a solution to Euler's equation:

$$\frac{d^6S(t)}{dt^6} = 0 \text{ или } \frac{d^4U(t)}{dt^4} = 0. \quad (6)$$

The equations (6) correspond to functionals:

$$\int_0^T \left(\frac{dU}{dt}\right)^2 dt \quad \text{или} \quad \int_0^T \left(\frac{d^3S}{dt^3}\right)^2 dt. \quad (7)$$

The following is an example of how to solve a problem in Maple.

$> restart;$ # *Пример использования полинома для задания перемещения (n=3)*

$> S := C6 \cdot t^5 + C5 \cdot t^4 + C4 \cdot t^3 + C3 \cdot t^2 + C2 \cdot t + C1 :$

$> V := diff(S, t) : U := diff(V, t) : dU := diff(U, t) :$

$> t := 0 : L1 := S = 0 : L2 := V = 0 : \; t := \frac{T}{2} : L3 := dU = 0 : L4 := U = 0 :$

$> t := T : \; L5 := S - L = 0 : L6 := V = 0 :$

$> solve(\{L1, L2, L3, L4, L5, L6\}, \{C1, C2, C3, C4, C5, C6\});$

$$\left\{C1 = 0, C2 = 0, C3 = \frac{5L}{T^2}, C4 = -\frac{10L}{T^3}, C5 = \frac{10L}{T^4}, C6 = -\frac{4L}{T^5}\right\}$$

$> C1 := 0 : C2 := 0 : C3 := \frac{5L}{T^2} : C4 := -\frac{10L}{T^3} : C5 := \frac{10L}{T^4} : C6 := -\frac{4L}{T^5} :$

$> t :='t': \; U := factor(U); \; V := factor(V); \; S := factor(S);$

$$U := \frac{10L(T - 2t)^3}{T^5} \qquad V := \frac{10Lt(-t + T)(T^2 - 2Tt + 2t^2)}{T^5}$$

$$S := \frac{Lt^2(5T^3 - 10T^2t + 10Tt^2 - 4t^3)}{T^5}$$

2. At n = 5 the following polynomial is used

$$S(t) = \sum_{j=1}^{8} C_i t^{i-1} \quad (8)$$

with addition of skew symmetry conditions

$$\frac{d^2U}{dt^2}\left(\frac{T}{2}\right) = 0, \qquad \frac{d^3U}{dt^3}\left(\frac{T}{2}\right) = 0. \quad (9)$$

After defining the constants, the expression to displacement takes the form:

$$S(t) = \frac{Lt^2(21T^5 - 70T^4t + 140T^3t^2 - 168T^2t^3 + 112Tt^4 - 32t^5)}{3T^7} \quad (10)$$

and respectively $V(t)=\dfrac{dS(t)}{dt}$, $\qquad U(t)=\dfrac{dV(t)}{dt}$.

Euler's equations in this case:

$$\frac{d^8 S(t)}{dt^8}=0 \text{ или } \frac{d^6 U(t)}{dt^6}=0. \qquad (11)$$

To these equations correspond to functionals:

$$\int_0^T \left(\frac{d^2 U}{dt^2}\right)^2 dt \quad \text{или} \quad \int_0^T \left(\frac{d^4 S}{dt^4}\right)^2 dt. \qquad (12)$$

An example of obtaining expressions for displacement, velocity and acceleration (in Maple) from universal analytic dependencies is given below.

> *restart*; # *Пример использования универсальных аналитических зависимостей для перемещения, скорости* и *ускорения*

> $S := \dfrac{L}{2\cdot T\cdot(n+1)}\cdot\left(\dfrac{(T-2\cdot t)^{n+2}}{T^{n+1}}+2\cdot n\cdot t+4\cdot t-T\right)$:

> $V := \dfrac{L\cdot(n+2)}{T\cdot(n+1)}\cdot\left(1-(T-2\cdot t)^{n+1}\,T^{-n-1}\right)$: $\quad U := \dfrac{L\,(2\,n+4)\left(\dfrac{T-2\,t}{T}\right)^n}{T^2}$:

> $n := 1$: $S := simplify(S)$; $V := simplify(V)$; $U := simplify(U)$;

> $S := \dfrac{L\,t^2\,(3\,T-2\,t)}{T^3}$: $V := \dfrac{6\,L\,t\,(T-t)}{T^3}$: $U := \dfrac{6\,L\,(T-2\,t)}{T^3}$:

> #--

> $n := 3$: $S := simplify(S)$; $V := simplify(V)$; $U := simplify(U)$;

> $S := \dfrac{L\,t^2\left(5\,T^3-10\,T^2\,t+10\,T\,t^2-4\,t^3\right)}{T^5}$: $V := \dfrac{10\,L\,t\left(T^3-3\,T^2\,t+4\,T\,t^2-2\,t^3\right)}{T^5}$:

> $U := \dfrac{10\,L\,(T-2\,t)^3}{T^5}$:

> # --

> $n := 5$: $S := simplify(S)$; $V := simplify(V)$; $U := simplify(U)$;

> $S := \dfrac{1}{3}\,\dfrac{L\,t^2\left(21\,T^5-70\,T^4\,t+140\,T^3\,t^2-168\,T^2\,t^3+112\,T\,t^4-32\,t^5\right)}{T^7}$:

> $V := \dfrac{14}{3}\,\dfrac{L\,t\left(3\,T^5-15\,T^4\,t+40\,T^3\,t^2-60\,T^2\,t^3+48\,T\,t^4-16\,t^5\right)}{T^7}$:

$U := \dfrac{14\,L\,(T-2\,t)^5}{T^7}$:

The criteria for action (according to Lagrange) and energy spent on the implementation of movement confirm a decrease in the energy intensity of optimal movement with an increase in the degree of polynomial. The action (according to Lagrange) decreases faster than the spent energy. If the action limit tends to $\dfrac{mL^2}{T}$, then the energy tends to $\frac{mL^2}{T^2}$. It can be argued that energy is an action per unit time.

As the polynomial degree increases at times t = 0 and t = T when moving from the initial quiescence state to the final quiescence state, the maximum acceleration values (by absolute value) increase asymptotically. In this case, the maximum velocity value at time t = $T/2$ decreases asymptotically. Graphs of displacement, velocity and acceleration at different polynomial degrees are given in Fig. 1, 2.

The possible limit energy is calculated (m = 1 kg, L = 1 m, T = 1 s) as:

$$W = \lim_{n\to\infty} 2m \int_0^{T/2} U(t)V(t)\,dt = 1.$$

The graph of energy versus polynomial degree $W(n)$ is shown in Fig. 3.

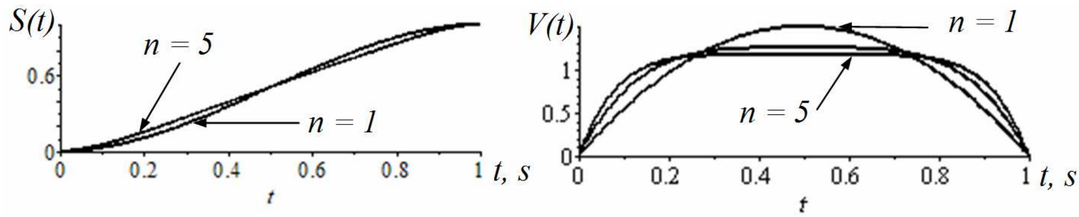

Fig. 1. Graphs of displacements $S(t)$ and velocities $V(t)$ at different polynomial degrees

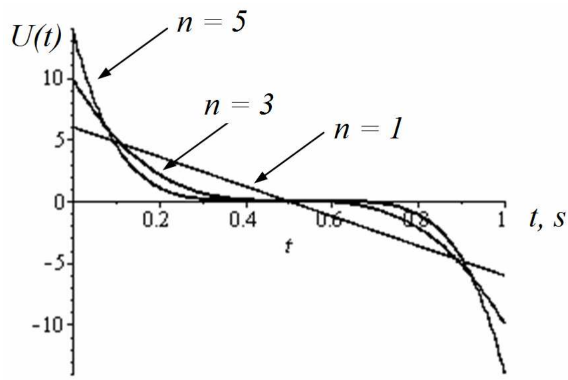

Fig. 2. Graphs of accelerations $U(t)$ at polynomial degrees n = 1, 3, 5

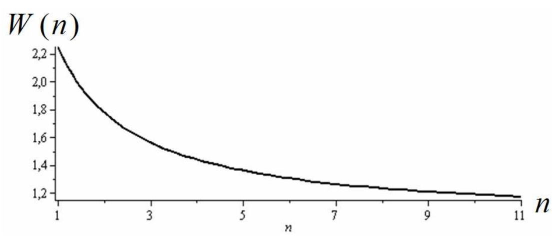

Fig. 3. Graph of $W(n)$

Kinetic energy tends to $\dfrac{mV^2(T/2)}{2} = m\left(\dfrac{L}{T}\right)^2$, but for all cases ($n$ = 1, 3, 5,...) the set purpose of motion is achieved in time T $\left(S(T) = L, \quad V(T) = 0\right)$.

The criterion of the power norm $\int\limits_0^T U^2 dt$ "compromises itself," i.e. increases. This is due to an increase in absolute values of $U(t)$.

Some generalized motion characteristics are presented in Table 1.

Tab. 1. Generalized motion characteristics

Degree of the poly-nomial n	Expressions for acceleration (control), velocity, displacement	at $t = 0$	at $t = \frac{T}{2}$	at $t = T$	Power norm $\int\limits_0^T U^2 dt$	Action (Lag-range form) $\int\limits_0^T V^2 dt$	Energy $2\int\limits_0^{T/2} UVdt$	Velocity $V\left(\frac{T}{2}\right) = \int\limits_0^{T/2} Udt$
1	$U(t) = \frac{6L(T-2t)}{T^3}$ $V(t) = \frac{6Lt(T-t)}{T^3}$ $S(t) = \frac{Lt^2}{T^3}(3T-2t)$	$\frac{6L}{T^2}$ 0 0	0 $\frac{3L}{2T}$ $\frac{L}{2}$	$-\frac{6L}{T^2}$ 0 L	$\frac{12L^2}{T^3}$	$\frac{1,2L^2}{T}$	$\frac{2,25L^2}{T^2}$	$\frac{1,5L}{T}$
3	$U(t) = \frac{10L}{T^5}(T-2t)^3$ $V(t) = \frac{10Lt(T-t)(2t^2-2Tt+T^2)}{T^5}$ $S(t) = \frac{Lt^2(5T^3-10T^2t+10t^2T-4t^3)}{T^5}$	$\frac{10L}{T^2}$ 0 0	0 $\frac{5L}{4T}$ $\frac{L}{2}$	$-\frac{10L}{T^2}$ 0 L	$\frac{14,286L^2}{T^3}$	$\frac{1,111L^2}{T}$	$\frac{1,562L^2}{T^2}$	$\frac{1,25L}{T}$
5	$U(t) = \frac{14L}{T^7}(T-2t)^5$ $V(t) = \frac{14Lt(3T^5-15T^4t+40T^3t^2)}{3T^7} +$ $+\frac{14Lt(-60T^2t^3 48Tt^4-16t^5)}{3T^7}$ $S(t) = \frac{Lt^2(21T^5-70T^4t+140T^3t^2)}{3T^7} +$ $+\frac{Lt^2(-168T^2t^3+112Tt^4-32t^5)}{3T^7}$	$\frac{14L}{T^2}$ 0 0	0 $\frac{7L}{6T}$ $\frac{L}{2}$	$-\frac{14L}{T^2}$ 0 L	$\frac{17,818L^2}{T^3}$	$\frac{1,077L^2}{T}$	$\frac{1,361L^2}{T^2}$	$\frac{1,167L}{T}$

Conclusions

1. The results of analytical calculations are summarized in a Table 1 that allows to analyze the properties of the designed controls.

2. Optimal control of acceleration-deceleration type by object during movement from initial to final state of quiescence for specified distance and time of motion are identified and investigated. In fact, with the increase in the degree of the

acceleration polynomial, an excess part of the energy for acceleration and deceleration is taken away.

3. The features of using the algorithm for solving the complete inverse problem of variational calculus (called reversive) are disclosed. With the increase in the degree of the control polynomial (translational acceleration), energy costs to achieve the purpose of movement are reduced.

4. In the practical implementation of this type of control, it is possible to replace it (for example, at $n \leq 7$) with energy-equivalent controls while maintaining the possibility of achieving the specified motion purpose.

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