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THE METHOD OF CALCULATING STRAIGHT-LINE CENTRALLY COMPRESSED BARS ON STABILITY

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Keywords: rod, stability, calculation, graphic-analytical method.

Abstract. When carrying out the design calculation of rectilinear centrally compressed rods for stability, the use of the coefficient of reduction of the main allowable stress leads to the need to use the method of successive approximations, which requires a large number of calculations, which is its significant disadvantage. Described in the article graphic-analytical calculation method is deprived of this drawback. To implement this method, it is necessary to analytically obtain the dependences of the reduction coefficient of the main allowable stress on flexibility and graphically determine their actual values as the coordinates of the intersection point of the graph of the obtained dependence and the graph constructed from reference values.

МЕТОД РАСЧЕТА ПРЯМОЛИНЕЙНЫХ ЦЕНТРАЛЬНО СЖАТЫХ СТЕРЖНЕЙ НА УСТОЙЧИВОСТЬ

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Ключевые слова: стержень, устойчивость, расчёт, графо-аналитический метод.

Аннотация. При проведении проектировочного расчёта прямолинейных центрально сжатых стержней на устойчивость использование коэффициента снижения основного допускаемого напряжения приводит к необходимости применения способа последовательных приближений, требующего большого количества вычислений, что является его значительным недостатком. Описываемый в работе графо-аналитический метод расчёта лишён данного недостатка. Для реализации данного метода необходимо аналитически получить зависимости коэффициента снижения основного допускаемого напряжения от гибкости и графического определения их действительных величин как координат точки пересечения графика полученной зависимости и графика, построенного по справочным значениям.

As it is known from the classical material resistance course, the slenderness criterion (hereinafter denoted by λ) can be used to classify bars as those of large, medium and small slenderness. When calculating the stability of the former, it is justified to apply the Euler formula, with medium ones the Yasinsky formula is engaged, however, small slenderness bars do not run the risk of losing stability as it must be preceded by the loss of their bearing capacity. These formulae allow to determine the value of normal stress σ_{kr}, corresponding to the critical force the excess of which will result in the loss of stability of the original form of balance of the homogeneous rectilinear centrally compressed bar. The graphical dependence of critical stress on the slenderness of the bar is featured in Figure 1. Here *Curve I* is relevant for large slenderness bars, thus, it matches the calculating correspondence $\sigma_{kr} = \pi^2 E/\lambda^2$, where $\lambda = \beta l/i$ is the bar slenderness; $i = \sqrt{J_{\min}/A}$ is the minimum inertia radius; $J_{\min}$ is the minimum axial moment of inertia of the cross section; A is the cross section area; β is the length reduction factor determined by the way of the bar fixation; and l is the bar length. *Curve II* limits the critical stress going to infinity at the reduction of the bar slenderness and it corresponds to the limiting value of σ_L (the yield point at compression $\sigma_{T.C.}$ for plastic materials or the break-down point for at compression $\sigma_{B.C.}$ for fragile ones). *Curve III* is described using the Yasinsky formula $\sigma_{kr} = a - b\lambda + c\lambda^2$, where *a*, *b*, *c* are the values determined by the bar material. λ_0 and $\lambda_{\lim}$ mean the values slenderness matching the boundary values/ thus, it is necessary to determine the slenderness of the bar first and then to select the proper formula to calculate the stress.

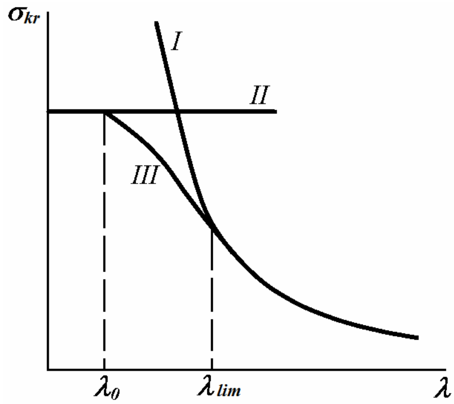

Fig. 1. Dependence between critical stresses and bar slenderness

The essence of the method of calculation on the basis of the main allowed stress is rather simple. It stems from the premise that with small slenderness bars the allowed compression stress equals $[\sigma]_C$, however, the increase in slenderness it has to be reduced by multiplying the reduction factor of the main allowed stress φ, the latter being less than 1. Numerical values of φ (that may be calculated using the correspondence $\varphi = \sigma_{kr} \cdot n_C / (n_y \cdot \sigma_L)$,where n_C is the load factor; n_y is the stability factor), depending on slenderness λ may be found in the reference books for different materials [1].

However, even using the pre-determined values of φ may trigger significant problems. Depending on the terms of a particular task, the calculation practice employs checking or designing calculation methods. As a rule, in checking calculations the employment of the φ value does not entail any difficulty. Knowing the geometrical parameters of the bar and the conditions of its fixation, it is possible to calculate the minimum inertia radius, slenderness and the allowed stress on stability. After that by comparing the stresses active in cross-sections of the bar with the compressing force F $(\sigma = F/A)$ with the allowed stress on stability, one can obtain relevant conclusions.

The case of designing calculations is different as when determining the area of the cross section, it is given that $\sigma = F/(\varphi A) \le [\sigma]_c$, i.e. $A \ge F/(\varphi[\sigma]_c)$, however, the value of φ is not given, so there are two unknown values in one formula, thus uncertainty occurs. To resolve the problem, it is necessary to resort to the successive approximation method [1], [2]. Setting the initial value of $\varphi_1 = 0{,}5...0{,}6$, we can calculate the necessary cross section area A_1. Then, taking into account the cross section type of the bar, we can calculate the minimum inertia radius i_1 and bar slenderness λ_1. Using the value of λ_1 allows to determine the value of the respective coefficient of allowed stress φ_1'. If the difference between φ_1' and φ_1 is deemed significant, it is necessary to repeat the calculations using the value of $\varphi_2 = (\varphi_1 + \varphi')/2$ at the second step of the calculation. New values of A_2, i_2, λ_2 and, consequently, φ_2' are determined. In case φ_2' and φ_2 diverge significantly again, the calculations are once again repeated using $\varphi_3 = (\varphi_2 + \varphi_2')/2$, etc.

Thus, the drawback of using the coefficient of the main allowed stress in designing calculations consists in the need to resort to the successive approximation method that requires a considerable amount of calculation. The suggested graphic-analytical method of calculation of rectilinear centrally compressed bars for stability is deprived of this drawback.

As has been mentioned before, the essence of the graphic-analytical method of calculating rectilinear centrally compressed bars for stability consists in finding the analytical function $\varphi(\lambda)$ of the coefficient φ and slenderness λ, the drawing of the graph of this function on the basis of the values of φ from λ, taken from references. In particular, Table 1 gives the respective parameters for steel with $\sigma_{\text{T.C.}} = 400\,MPa$ [3].

Tab. 1.Values of φ depending on slenderness λ

λ	0	10	20	30	40	50	60	70	80	90	100	110
φ	1,000	0,982	0,949	0,905	0,854	0,796	0,721	0,623	0,532	0,447	0,369	0,306
λ	120	130	140	150	160	170	180	190	200	210	220	
φ	0,260	0,223	0,195	0,171	0,152	0,136	0,123	0,111	0,101	0,093	0,086	

In [4] it was shown how to use the graphic-analytical calculation method for bars having a simple cross-section type (a circle or a square), which allows to obtain rather a plain dependence between the cross-section area *A* and the axial inertia moment $J_{\min}$. For bars having a more complex cross section where it is impossible to use only one parameter to determine the area (diameter, length of side, etc.) this method does not seem appropriate at first glance. However, this method may be applied in these cases as well, in case there is one-to-one association between the parameters characterizing the dimensions of this cross section. The example may be found in the rectangular bar with the cross section having dimensions $b \times h$ with a fixed end, centrally loaded by the force applied whose value is *F*, as is shown in Figure 2. Here $A = bh$; $J_{\min} = hb^3/12$; $i = \sqrt{J_{\min}/A} = b/\sqrt{12}$; $\lambda = \beta l/i = \sqrt{12}\beta l/b$. If we take $h = 2b$ in this example, then $\sqrt{A} = \sqrt{2}b$ and $\lambda = \sqrt{24}\beta l/\sqrt{A}$, hence, $A = 24\beta^2 l^2/\lambda^2$. On the other hand,

$A \geq F/(\varphi[\sigma]_c)$. Comparing the last two statements, we conclude the formula for the minimum value of the cross section area of the bar: $\varphi = F\lambda^2/24\beta^2 l^2[\sigma]_c$.

For the bar in question $\beta = 2$ (rigidly fixed one end of the bar). If we take the value of $F = 500\,\kappa N$, and $l = 1\,m$. Using the data from Table 1, we can build the graph of $\varphi(\lambda)$ and the graph of dependence of $\varphi = F\lambda^2/24\beta^2 l^2[\sigma]_c$. We obtain the values of φ and λ, as the coordinates of the point where these two graphs intersect. The respective constructs are featured in Figure 3. The value of $\lambda \approx 126$ obtained in Figure 3 is further used in formula $A = 24\beta^2 l^2/\lambda^2$. Taking into account that with the chosen rectangular cross section $b = \sqrt{A/2}$, we obtain $b = 55\,mm$.

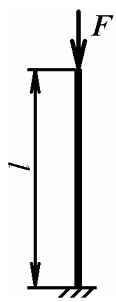

Fig. 2. Bar design scheme

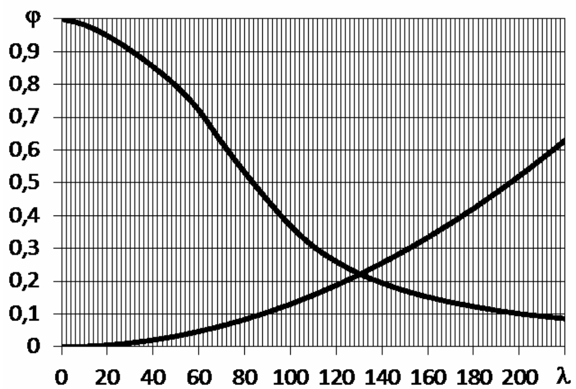

Fig. 3. Determining the coefficient of reducing allowed stress for a rectangular cross section

Let us consider a rectangular bar having a cross section shaped as a right triangle with legs b and h. Here [3] $J_{\min} = (J_x + J_y)/2 - \sqrt{((J_y - J_x)/2)^2 + J_{xy}^2}$, where J_x, J_y, J_{xy} are axial and centrifugal inertia moments relative to central axis x and y that are parallel to the legs. By way of example, let us take $h = 2b$. Then $J_{\min} = (2 - \sqrt{13})b^4/36$. Considering that $A = bh/2 = b^2$, and $i = \sqrt{J_{\min}/A} = \sqrt{5 - \sqrt{13}}\,b/6$, then $\lambda = \beta l/i = 6\beta l\Big/\left(\sqrt{5 - \sqrt{13}}\,b\right)$, $\lambda = 6\beta l\Big/\sqrt{A(5 - \sqrt{13})}$. Hence $A = 36\beta^2 l^2/((5 - \sqrt{13})\lambda^2)$. On the other hand, $A \geq F/(\varphi[\sigma]_c)$. Comparing the last two statements, we can conclude the formula for the minimum value of the square of the cross section: $\varphi = F\lambda^2(5 - \sqrt{13})/36\beta^2 l^2[\sigma]_c$. We build the graph illustrating the latter dependence using the data from Table 1, as is shown in Figure 4. The value of $\lambda \approx 131$ obtained from Figure 4 is then inserted in $A = 36\beta^2 l^2/((5 - \sqrt{13})\lambda^2)$. Considering that in the chosen rectangular cross section $b = \sqrt{A}$, we obtain the value of $b = 78\,mm$.

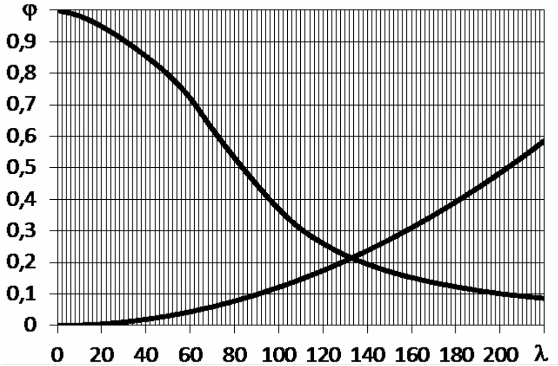

Fig. 4. Determining the coefficient of reducing the allowed stress for the bar with a triangle cross section

The application of the calculation method described is justified by the outcomes of the numerical solutions using the Nastran software. Figures 5 and 6 illustrate the corrugated bars having rectangular and triangle cross sections respectively, loaded with the compressed force of 600 *кN*. The scales on the right reflect full shifts. For better visibility Figures 7 and 8 present the graphs of longitudinal movements of δ along the original axes in the undeformed state of unfixed bar ends obtained by conducting non-linear calculations using Nastran. A significant growth in the bend begins with the value of force at approximately 500 *kN*, which proves the correctness of the outcomes of the calculations obtained using the graphic-analytical method.

Thus the suggested method of calculation can be applied not only to bars having a simple cross section determined by one parameter (diameter, side length, etc.), but for more complex cross sections. The key point is the presence of one-to-one association between parameters characterizing their dimensions.

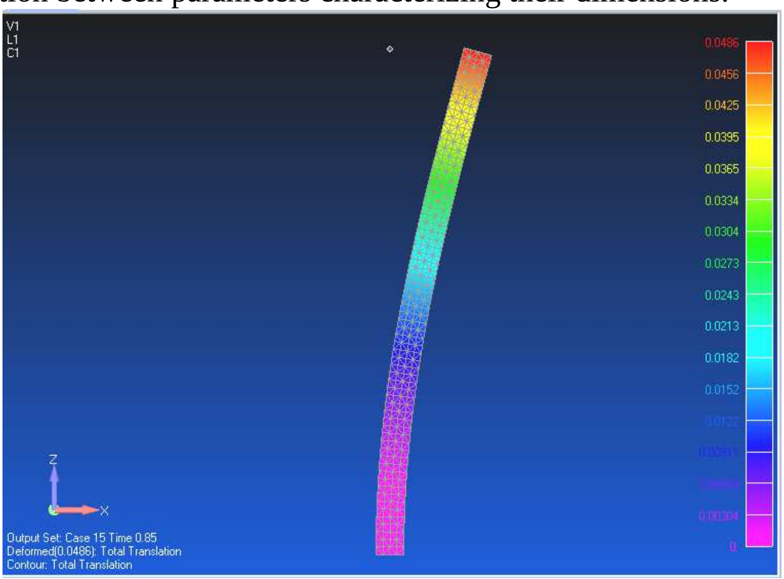

Fig. 5. Outcomes of calculations using the Nastran software for a bar with a rectangular cross section

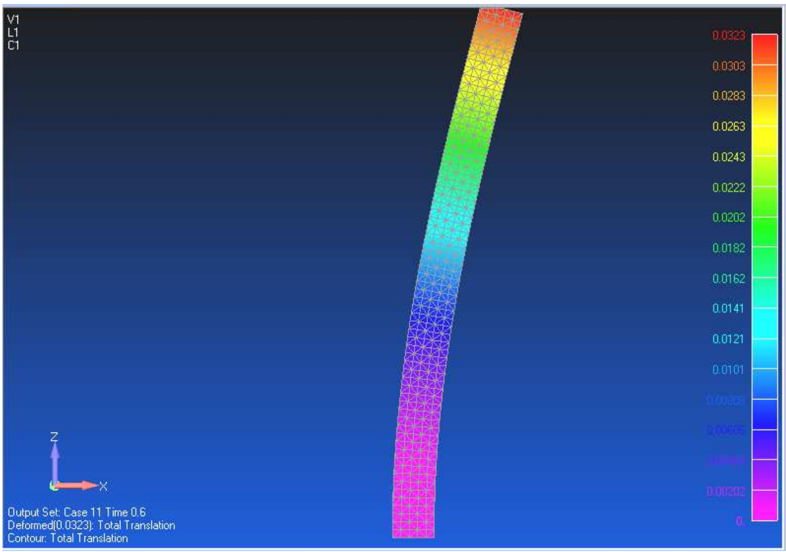

Fig. 6. Outcomes of calculations using the Nastran software for a bar with a triangle cross section

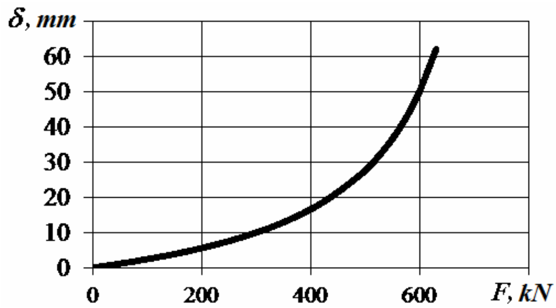

Fig. 7. Dependence between the moving of the free end of the bar with a rectangular cross section and the compressing force

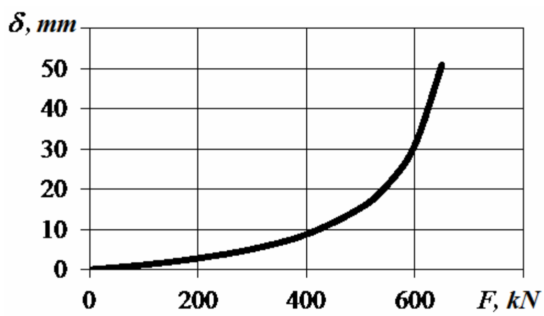

Fig. 8. Dependence between the moving of the free end of the bar with a triangle cross section and the compressing force

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