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SYSTEM BASES OF FORMATION OF A SET OF STANDARD LINKS IN APPLICATIONS OF STRUCTURAL MATHEMATICAL MODELING METHODS TO PROBLEMS OF DYNAMICS OF TECHNOLOGICAL AND TRANSPORT MACHINES

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Keywords: technical object, mechanical oscillatory system, structural mathematical model, elements, transfer function, feedback.

Abstract. The possibilities of developing system approaches and system analysis in building the foundations of the method of structural mathematical modeling in the application to the problems of dynamics of technical objects operating under conditions of intense dynamic loading and vibration interaction of machine elements are considered. The purpose of the research is to develop a system of basic concepts of structural mathematical modeling, develop principles for forming a set of typical elementary links and rules for their switching in solving problems of forming and controlling dynamic states of mechanical oscillatory systems, considered as calculation schemes of technical objects. Methods of oscillation theory, integral transformations, and automatic control theory are used. The basic methodological basis for using the analytical apparatus of the automatic control theory for evaluating the dynamic properties of technical objects in the problems of vibration interactions of system elements has been developed.

СИСТЕМНЫЕ ОСНОВЫ ФОРМИРОВАНИЯ НАБОРА ТИПОВЫХ ЗВЕНЬЕВ В ПРИЛОЖЕНИЯХ МЕТОДОВ СТРУКТУРНОГО МАТЕМАТИЧЕСКОГО МОДЕЛИРОВАНИЯ К ЗАДАЧАМ ДИНАМИКИ ТЕХНОЛОГИЧЕСКИХ И ТРАНСПОРТНЫХ МАШИН

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Ключевые слова: технический объект, механическая колебательная система, структурная математическая модель, элементы, передаточная функция, обратная связь.

Аннотация. Рассматриваются возможности развития системных подходов и системного анализа в построении основ метода структурного математического моделирования в приложении к задачам динамики технических объектов, работающих в условиях интенсивного динамического нагружения и вибрационных взаимодействие элементов машин. Цель исследования заключается в разработке системы основных понятий структурного математического моделирования, разработке принципов формирования набора типовых элементарных звеньев и правил их коммутации в решении задач формирования и управления динамическими состояниями механических колебательных систем, рассматриваемых как расчетные схемы технических объектов. Используются методы теории колебаний, интегральных преобразований и теории автоматического управления. Разработана принципиальная методологическая основа использования аналитического аппарата теории автоматического управления для оценки динамических свойств технических объектов в задачах вибрационных взаимодействий элементов системы.

Introduction. Technical objects, such as vehicles and technological vibration machines operating under intense dynamic loads, require increased attention to the assessment of the features of dynamic States both at the stages of preliminary research, design and calculation, and during operation, involving the assessment, control and management of parameters of dynamic States.

The problems of machine dynamics are diverse and reflect attention to the effects of external disturbances that lead to vibration movements and interactions of machine elements, an increase in dynamic loads on the corresponding components, aggregates and devices, and a decrease in performance indicators of dynamic quality. [1-4].

Methods of structural mathematical modeling allow us to develop and support the development of specific ways and means to reduce the level of impacts on technical objects, their working bodies, equipment and equipment. One of the most developed directions in this field can be considered the development of a methodological basis for solving problems of vibration protection and vibration isolation of objects [5-8].

Structural mathematical modeling, as a methodological position, is most promising on models that reflect the features of dynamic States of mechanical oscillatory systems, in relation to which methods of constructing differential equations of motion based on Lagrange formalism are applied, followed by the use of methods of integral Laplace transformations and obtaining operator forms that allow the transition to structural mathematical models in the form of structural schemes that are equivalent in dynamic terms, automatic control systems [9-13].

The presence of structural schemes makes it possible to use the analytical apparatus of automatic control theory to evaluate the dynamic properties of systems. The possibilities of such approaches based on the methods of systems theory and system analysis are considered in specific applications in [3, 14-16].

This article is devoted to the development and detailing of ideas about the basics of the structural mathematical modeling method in problems of dynamics of machines operating under vibration loads.

I. Some general provisions. The problems of dynamics of many technical objects, which are characterized by working under dynamic loads of external and internal nature, considered in the calculation schemes as mechanical oscillatory systems with one or more degrees of freedom, in many cases are reduced to the problems of vibration protection and vibration isolation of a certain mass-inertia element or block within the framework of representations of a system that has a certain structure, dynamic connections and constituent elements that generally form an idea of the corresponding system [3, 6-8].

The structural theory of vibration-proof systems has the possibility of using structural analogues of mathematical models in the form of structural schemes of dynamically equivalent automatic control systems (ACS). In control theory, as well as in vibration protection structures, typical elementary links are used. Note that the transfer function of the ACS, which has the form of an arbitrary polynomial, can be decomposed into prime factors, that is, it is represented by transfer functions of simpler elementary typical links.

With all the common approaches, the construction of additional circuits introduced into the structure of vibration protection systems (VPS) has its own specifics and requires taking into account a number of features.

The transfer function of the additional feedback circuit in a vibration protection system (VPS) is defined in the general case by the expression

$$W_{add}(p)=\frac{a_0+a_1p+...+a_np^n}{b_0+b_1p+...+b_mp^m}, \quad (1)$$

where $n,m-$ are natural numbers $(n\le m), a_i, b_i-$coefficients that depend on the design features of the VPS$\left(i=1,\overline{n};\ j=1,\overline{m}\right)$. In accordance with (1), the elementary links of the VPS can be found by simplifying $W_{add}(p)$ through the corresponding "zeros" a_i, b_j in a certain order.

The emerging problem is that it is necessary to take into account the boundaries of the "closing" of the two approaches developed in the theory of VPS and the theory of automatic control, focused on the consideration of elementary links [17-19].

II. Comparative analysis of the elements' capabilities. In the theory of automatic control (TAC), the choice of typical links is based, to a large extent, on the idea of a certain set of typical links, due to the tradition and connection with Laplace integral transformations:

$$k,p,\frac{1}{p},Tp\pm1,\frac{1}{Tp\pm1},T^2p^2\pm2\xi Tp+1,\frac{1}{T^2p^2\pm2\xi Tp+1}, \quad (2)$$

where $p=j\omega$ is a complex variable $(j=\sqrt{-1})$.

Note that the coefficient $\boldsymbol{k}$ in the theory of automatic control [17-19] is called the transfer coefficient, T - the time constant, and $\xi(0<\xi<1)$ - the damping coefficient. From the above positions in the understanding of elementary links in the expression (2), we can see that the first three links $k,\ p,\ \frac{1}{p}$ are elementary, and the rest can be obtained by using the rules of sequential and parallel connections, as is customary in the theory of automatic control. A diagram of the ratio of conversion rules is shown in fig. 1. Note the essential feature of the mechatronics VPS, which is determined by the fact that an additional connection (or a chain) has the dimension of the input signal level in the offset, and the output is a force, that is considered essentially a mechanical circuit consisting of elements of the same type. There are five such elements, but a pure lag link can be added to them under the same conditions. Such elements are combined to form more complex structures according to the rules of sequential and parallel addition of springs [17-20].

The transfer function (1), in our case, refers to an additional mechanical circuit introduced in a certain way into the block diagram of a dynamically equivalent ACS. The complexity of this circuit determines the structure0, the number of elementary links, and the rules for switching them, as shown in figure 1.

When considering the block diagram, the transfer function of the VPS as a whole is determined, which gives an expression that has, in general, the same type as the expression (1). But in two cases, these expressions, despite their external similarity, have different physical meaning, given that the element base of the VPS is formed within the framework of additional feedback.

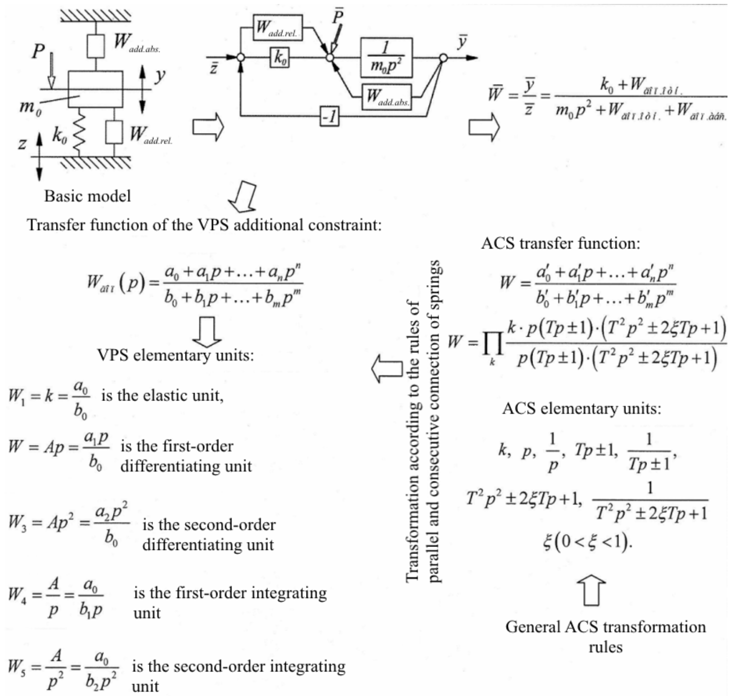

Fig. 1. Diagram explaining the relationship between the conversion rules in the additional feedback circuit and the ACS block diagram

III. General methodological provisions on the structural approach. The basis of the vibration protection system is a mechanical oscillatory structure. Its simplest form is an object of mass *m* and an elastic element *k*. Such a system can be called a basic model. It corresponds to a dynamically equivalent automatic control system (Fig. 2, a, b).

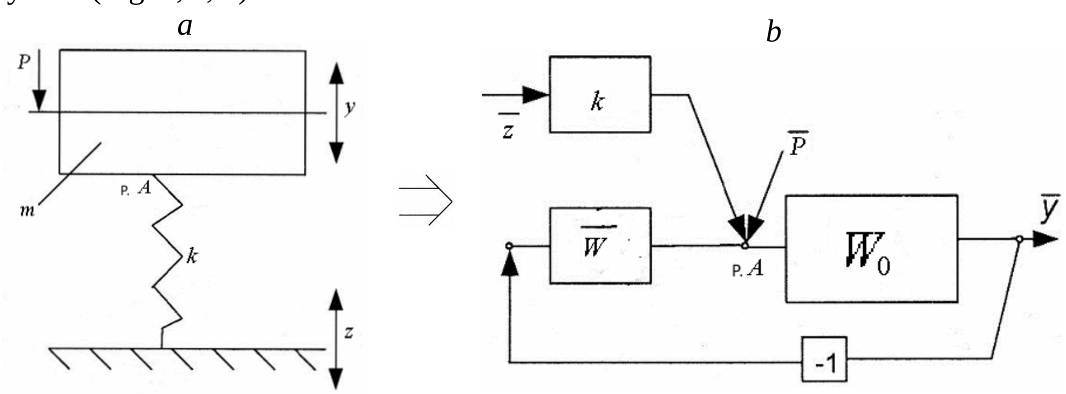

Fig. 2. The base model VPS (a) and its block diagram (b)

The block diagram gives an idea of the location and role of the elastic element, which is fed to the input "displacement" $\bar{y}$. The output of the link with the transfer function $W_{\text{add}} = k$ is the force. This force is applied to the input of the second link with the transfer function $W_0 = \frac{1}{mp^2}$. Such a link in the block diagram is a "summator", since all forces are gathered at the input.

This link performs a basic function and is not included in the number of elementary links and has a different transfer function in dimension, since the output signal is an displacement $\bar{y}$. In such a system of minimal complexity, the link $W_{add} = k$ creates a force of elastic interaction. From the point of view of the theory of automatic control (TAC), the mentioned link is called an amplifying one. The second base link is a second-order integrating link that "transforms" the elastic force into an offset. Both schemes in Fig. 2 a and b correspond to the same differential equation:

$$m\ddot{y} + ky = kz + Q(t). \tag{3}$$

The block diagram in Fig. 2 b is a graphical form of representation of the differential equation (3). Obtaining equation (3) can be implemented on the basis of the Dalembert principle or by using the Lagrange equation. For us, an important circumstance is the choice of a "key" element in the basic model – an object of mass *m*. More precisely – the "key" element is the material point mass *m*, relative to which the kinetostatics equation is made. An elastic element with a coefficient of rigidity is thus an elementary link, and the other two links with transfer functions W_1 and W_2 - create a basic model of minimal complexity: $W' = k$ it is included in an expanded set of elementary links of vibration protection systems.

We will assume that the physical or constructive-technical form of implementation of typical links is feasible on the basis of links of passive and active nature. The latter is related to the use of mechanisms driven by external energy sources in the VPS [14-16].

Characteristic of the approach developed in the article is the perception of these typical elementary links, homogeneous, in the sense that they all have a transfer function that represents (in the field of Laplace transformations) the ratio of the image of the output signal to the input signal. In this case, the input signal is an offset, and the output signal is a force factor of the corresponding type. In this sense, typical elementary links are compatible, and their connection is determined by the rules of parallel and sequential connections, as two elastic elements (or links). Figure 3 shows the ratio of traditional and extended sets of elements.

Since typical elements are homogeneous (in the above meaning), transfer functions of typical elements of the second level can be obtained from a combination or combination of two different elements, which consist of basic elements using the rules of "parallel and sequential" assembly [4].

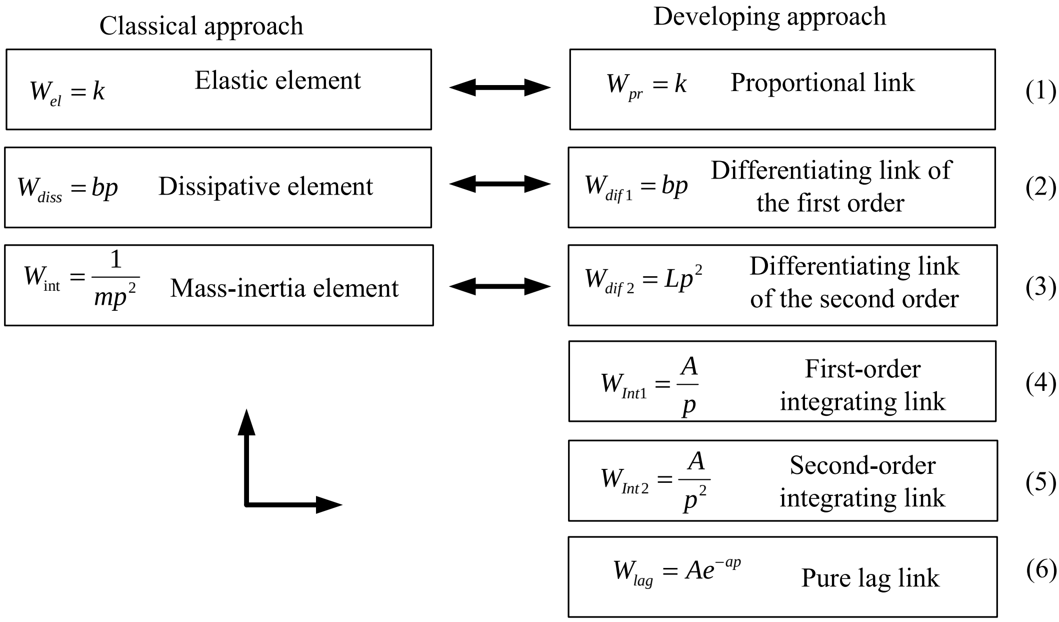

Fig. 3. The ratio of traditional and advanced approaches of model elements VPS

IV. Features of the rules for connecting VPS links. Let's look at a number of examples of connections that are not of the same type, but of connections of elements of different types. Let's present the transfer functions of the second-level links using information about possible combinations when the typical elements of the first level are connected in series and parallel (Table 1).

Further increase in the complexity of combinational construction leads to the concept of a mechanical chain formed from connected typical elementary links based on the mentioned rules of structural transformations. Note that the additional feedback taken by itself is quite rationally transformed into compact structures based on the concepts of bipolars, which was given in [20].

Tab. 1. Possible types of standard second-level links

№	Types of links	Parallel connection	Serial connection
1.	$W_1 = k$, $W_2 = Ap$	$W' = k + Ap$	$W'' = \frac{kAp}{Ap + k}$
2.	$W_1 = k$, $W_2 = Lp^2$	$W' = k + Lp^2$	$W'' = \frac{kLp^2}{Lp^2 + k}$
3.	$W_1 = k$, $W_2 = \frac{A}{p}$	$W' = \frac{A + kp}{p}$	$W'' = \frac{Ak}{A + kp}$
4.	$W_1 = k$, $W_2 = \frac{A}{p^2}$	$W' = \frac{p^2k + A}{p^2}$	$W'' = \frac{Ak}{A + kp^2}$
5.	$W_1 = k$, $W_2 = Ae^{-ap}$	$W' = kAe^{-ap}$	$W'' = \frac{Ak}{ke^{-ap} + A}$

Tab. 1. Possible types of standard second-level links

№	Types of links	Parallel connection	Serial connection
6.	$W_1 = bp$, $W_2 = Lp^2$	$W' = bp + Lp^2 = p(b + Lp)$	$W'' = \frac{bLp^2}{Lp + b}$
7.	$W_1 = bp$, $W_2 = \frac{A}{p}$	$W' = \frac{A + bp^2}{p^2}$	$W'' = \frac{Abp}{bp^2 + A}$
8.	$W_1 = Lp^2$, $W_2 = \frac{A}{p}$	$W' = \frac{Lp^3 + A}{p}$	$W'' = \frac{ALp^2}{Lp^3 + A}$
9.	$W_1 = bp$, $W_2 = Ae^{-ap}$	$W' = bp + Ae^{-ap}$	$W'' = \frac{bpAe^{-ap}}{bp + Ae^{-ap}}$
10.	$W_1 = \frac{A}{p^2}$, $W_2 = Lp^2$	$W' = \frac{Lp^4 + A}{p^2}$	$W'' = \frac{ALp^2}{Lp^4 + A}$
11.	$W_1 = Lp^2$, $W_2 = Ae^{-ap}$	$W' = Lp^2 + Ae^{-ap}$	$W'' = \frac{Lp^2 Ae^{-ap}}{Lp^2 + Ae^{-ap}}$
12.	$W_1 = \frac{A_1}{p}$, $W_2 = \frac{A_2}{p^2}$	$W' = \frac{A_1 p + A_2}{p^2}$	$W'' = \frac{A_1 A_2}{p(A_1 p + A_2)}$
13.	$W_1 = \frac{A_2}{p^2}$, $W_2 = Ae^{-ap}$	$W' = \frac{A_2 + Ap^2 e^{-ap}}{p^2}$	$W' = \frac{A_2 A}{A_2 e^{-ap} + p^2}$

Conclusion. A characteristic circumstance that I would like to note is that by choosing a transfer function of a particular form, from the expression (1), it is simultaneously possible to obtain a preliminary assessment of dynamic properties, followed by a choice of methods for the physical implementation of such a chain. In this plan, at least three directions can be considered: a combination of elementary links of an extended standard set; combinations through connections of standard links based on structural transformation rules; search for physical implementations of mechanical systems that have a transfer function of a particular type from expression (1) based on various types of mechanisms or servos.

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