https://doi.org/10.26160/2474-5901-2019-17-11-21

DETERMINING THE STRESS STATE OF A LAYER ON A RIGID BASE WEAKENED BY SEVERAL LONGITUDINAL CYLINDRICAL CAVITIES

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Keywords: cylindrical cavities in the layer, Lame's equation, generalized Fourier method, infinite systems of linear algebraic equations.

Abstract. The spatial problem of the theory of elasticity is solved for a layer with several infinite cylindrical cavities disjoint to each other and the surface of the layer. Stress values are preset for the cavities and for the upper boundary of the layer; displacements are preset on the lower boundary of the layer. The problem is solved using the generalized Fourier method with respect to the system of Lame's equations. If the boundary conditions are satisfied, we are led to infinite systems of linear algebraic equations that are solved by the reduction method. As a result, values of displacements and stresses at various points of the elastic layer are obtained. A computational investigation is carried out for a concrete layer adhered to a rigid base and weakened by two unloaded cavities. A normal stress value is preset on the upper boundary of the layer. The analysis of the stress-strain state of the layer in the vicinity of the load application, as well as in the vicinity of the left cavity located closer to the load, is carried out. It is compared to the option when the second cavity is absent. The proposed method can be used to calculate structures and parts with similar design models, and the stress state analysis can be made to select the geometric characteristics of the designed structure.

ОПРЕДЕЛЕНИЕ НАПРЯЖЕННОГО СОСТОЯНИЯ СЛОЯ НА ЖЕСТКОМ ОСНОВАНИИ, ОСЛАБЛЕННОГО НЕСКОЛЬКИМИ ПРОДОЛЬНЫМИ ЦИЛИНДРИЧЕСКИМИ ПОЛОСТЯМИ

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Ключевые слова: цилиндрические полости в слое, уравнение Ламе, обобщенный метод Фурье, бесконечные системы линейных алгебраических уравнений.

Аннотация. Решена пространственная задача теории упругости для слоя с несколькими бесконечными цилиндрическими полостями, непересекающимися между собой и поверхностью слоя. На полостях и на верхней границе слоя заданы напряжения, на нижней границе слоя заданы перемещения. Решение задачи получено при помощи обобщенного метода Фурье относительно системы уравнений Ламе. Удовлетворение граничным условиям приводит к бесконечным системам линейных алгебраических уравнений, которые решены методом редукции. В результате получены перемещения и напряжения в различных точках упругого слоя. Численное исследование проведено для слоя бетона, сцепленного с жестким основанием и ослабленного двумя полостями, свободными от нагрузки. На верхней границе слоя задано нормальное напряжение. Дан анализ напряженно - деформированного состояния слоя в близи приложения нагрузки, а также в окрестности левой полости, расположенной ближе к нагрузке. Проведено сравнение с вариантом, когда вторая полость отсутствует. Предложенный метод может использоваться для расчета конструкций и деталей с подобными расчетными схемами, а анализ напряженного состояния для подбора геометрических характеристик проектируемой конструкции.

1. Introduction

When designing various kinds of structures, machine parts and mechanisms design models in the form of a layer with circular cylindrical cavities are widespread. Therefore, a lot of articles are devoted to this topic.

So in the articles [1-5], problems for a layer with cavities perpendicular to its boundaries were considered. However, the methods used for crosscut cavities cannot be applied to the problems with longitudinal cavities.

The problem concerning heat shock for an infinite body with a cylindrical cavity dealing with the fractional order generalized theory, whose solution was obtained applying the Laplace transform theorem, was studied in the article [6].

The problems for a layer with one longitudinal cavity or inclusion were considered in the article [7] where based on the solution transformation in Fourier series and the reflection method stationary problems of shear waves diffraction were solved.

In the article [8], on the basis of the method of images, in two-dimensions, numerical and analytical calculations of diffraction scattering of symmetric normal waves of longitudinal shear for a layer with a cylindrical cavity were made.

The articles [9, 10] were devoted to determining the stress state of a finite cylinder and are based on the method of superposition of solutions and transformation in Fourier and Dini series.

In the articles [11, 12], based on the finite element method in three-dimensions, stresses and strain concentrations of round and elliptical holes in the plates of finite thickness under uniaxial tensile loading were considered.

All the methods mentioned above do not allow to solve the static problem with several cylindrical cavities in the layer; it is proposed to solve such problems applying the generalized Fourier method [13].

Based on this method, the problem for a layer with a spherical cavity, which is stretched by radial forces to infinities [14], problems for a half-space with a cylindrical cavity or inclusion [15, 16], a problem for a cylinder with cylindrical inclusions [17] as well as a problem for a layer with elastic inclusion are solved [18].

There are no precise and analytical and numerical methods for a layer with several cavities in the spatial option, although they can be found in design models. Therefore, the problem of solving such problems is relevant. In this work, the solution for the problem is obtained on the basis of new theorems of addition of vector solutions of the Lame's equation [19].

2. Problem formulation

In an elastic homogeneous layer, there are *N* cylindrical cavities parallel to its surfaces. Their radius is *Rp*, where *p* is the cylinder number, $p = 1, 2, ..., N$. Each cavity will be considered in a local cylindrical system of coordinates (ρ_p, φ_p, z), the boundaries of the layer will be considered in Cartesian coordinate system (x, y, z) combined with the axis of the cylinder with number q (Fig. 1). The upper boundary of the layer is at the distance $y = h$, the lower one is at the distance $y = -\tilde{h}$.

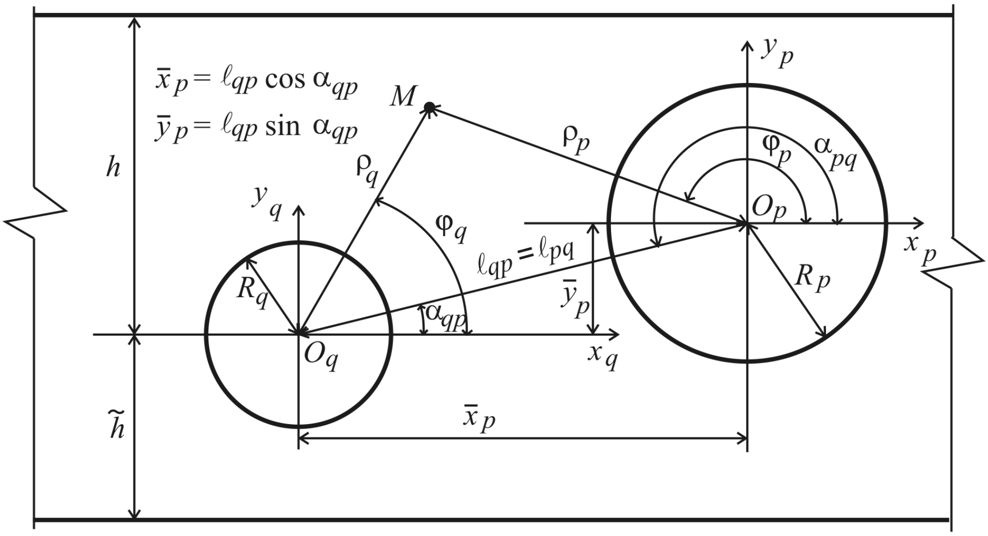


Fig. 1. Layer with cylindrical cavities

It is necessary to find a solution for the Lame's equation provided that the following values are preset: stresses $F\vec{U}(x,z)\big|_{y=h} = \vec{F}_h^0(x,z)$ on the upper boundary of the layer; displacements $\vec{U}(x,z)\big|_{y=-\tilde{h}} = \vec{U}_{\tilde{h}}^0(x,z)$ on the lower boundary of the layer, stresses $F\vec{U}_p(\varphi_p,z)\big|_{\rho_p=R_p} = \vec{F}_p^0(\varphi_p,z)$ on the surface of the cylinders, where $\vec{U}$ is the displacement vector;

$$F\vec{U} = 2\cdot G\cdot\left[\frac{\sigma}{1-2\cdot\sigma}\vec{n}\cdot\operatorname{div}\vec{U} + \frac{\partial}{\partial n}\vec{U} + \frac{1}{2}\left(\vec{n}\times\operatorname{rot}\vec{U}\right)\right]$$ is the stress vector;

σ - Poisson's ratio; $\vec{n}$ - unit normal vector to the surface of the layer or cylinder; G - the shear modulus;

$$\begin{aligned}
&\vec{F}_h^0(x,z) = \tau_{yx}^{(h)}\vec{e}_1^{(1)} + \sigma_y^{(h)}\vec{e}_2^{(1)} + \tau_{yz}^{(h)}\vec{e}_3^{(1)}\\
&\vec{U}_{\tilde{h}}^0(x,z) = U_x^{(\tilde{h})}\vec{e}_1^{(1)} + U_y^{(\tilde{h})}\vec{e}_2^{(1)} + U_z^{(\tilde{h})}\vec{e}_3^{(1)}\\
&\vec{F}_p^0(\varphi_p,z) = \sigma_\rho^{(p)}\vec{e}_1^{(2)} + \tau_{\rho\varphi}^{(p)}\vec{e}_2^{(2)} + \tau_{\rho z}^{(p)}\vec{e}_3^{(2)}
\end{aligned}, \tag{1}$$

known functions; $\vec{e}_j^{(k)}$, (j = 1, 2, 3) - Cartesian unit vectors (k = 1) and cylindrical (k = 2) systems of coordinates.

We consider functions (1) to be rapidly decreasing from the origin of coordinates along the z-coordinate for the cylinders and along the x and z-coordinates for the layer boundaries.

3. The solution method

We choose basic solutions for the Lame's equation taking into account the introduced systems of coordinates, in the form [13]:

$$\vec{u}_k^{\pm}(x,y,z;\lambda,\mu) = N_k^{(d)} e^{i(\lambda z + \mu x) \pm \gamma y};$$
$$\vec{R}_{k,m}(\rho_p, \varphi_p, z; \lambda) = N_k^{(p)} I_m(\lambda \rho_p) e^{i(\lambda z + m\varphi_p)}; \quad (2)$$
$$\vec{S}_{k,m}(\rho_p, \varphi_p, z; \lambda) = N_k^{(p)} \left[(\operatorname{sign}\lambda)^m K_m(|\lambda|\rho_p) \cdot e^{i(\lambda z + m\varphi_p)} \right]; k = 1,2,3;$$
$$N_1^{(d)} = \frac{1}{\lambda}\nabla;\ N_2^{(d)} = \frac{4}{\lambda}(\sigma - 1)\vec{e}_2^{(1)} + \frac{1}{\lambda}\nabla(y\,\cdot);\ N_3^{(d)} = \frac{i}{\lambda}\operatorname{rot}(\vec{e}_3^{(1)}\,\cdot);$$
$$N_1^{(p)} = \frac{1}{\lambda}\nabla;\ N_2^{(p)} = \frac{1}{\lambda}\left[\nabla\left(\rho\frac{\partial}{\partial\rho}\right) + 4(\sigma - 1)\left(\nabla - \vec{e}_3^{(2)}\frac{\partial}{\partial z}\right)\right];$$
$$N_3^{(p)} = \frac{i}{\lambda}\operatorname{rot}(\vec{e}_3^{(2)}\,\cdot);\ \gamma = \sqrt{\lambda^2 + \mu^2},\ -\infty < \lambda, \mu < \infty,$$

where $\vec{e}_j^{(r)}$, $(j = 1,2,3)$ - unit vectors of Cartesian (r = 1) and cylindrical (r = 2) systems of coordinate. σ - Poisson's ratio; $I_m(x)$, $K_m(x)$- modified Bessel function, $\vec{R}_{k,m}$, $\vec{S}_{k,m}$- respectively internal and external solutions for the Lame's equation for cylinder; $\vec{u}_k^{(-)}$, $\vec{u}_k^{(+)}$- the Lame's equation solutions for layer.

The solutions for the problem is presented as follows

$$\vec{U} = \sum_{p=1}^{N}\sum_{k=1}^{3}\int_{-\infty}^{\infty}\sum_{m=-\infty}^{\infty} B_{k,m}^{(p)}(\lambda)\cdot\vec{S}_{k,m}(\rho_p, \varphi_p, z; \lambda)d\lambda +$$
$$\sum_{k=1}^{3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(H_k(\lambda,\mu)\cdot\vec{u}_k^{(+)}(x,y,z;\lambda,\mu) + \tilde{H}_k(\lambda,\mu)\cdot\vec{u}_k^{(-)}(x,y,z;\lambda,\mu)\right)d\mu d\lambda, \quad (3)$$

where $\vec{S}_{k,m}(\rho,\varphi,z;\lambda)$, $\vec{u}_k^{(+)}(x,y,z;\lambda,\mu)$ and $\vec{u}_k^{(-)}(x,y,z;\lambda,\mu)$ are basic solutions determined by formulae (2) and the unknown functions $H_k(\lambda,\mu)$, $\tilde{H}_k(\lambda,\mu)$ and $B_{k,m}^{(p)}(\lambda)$ should be found taking into account the boundary conditions (1).

For transition from one coordinate system to another (Fig.1) on the base of [20] transition formulae for the basic solutions are obtained.

– for transition from solutions $\vec{S}_{k,m}$ of the cylindrical coordinate system to the layer solutions $\vec{u}_k^{(-)}$ (where y>0) and $\vec{u}_k^{(+)}$ (where y<0)

$$\vec{S}_{k,m}(\rho,\varphi,z;\lambda) = \frac{(-i)^m}{2}\int_{-\infty}^{\infty}\omega_{\mp}^m \cdot e^{-i\mu\bar{x}_p \pm \gamma\bar{y}_p}\cdot\vec{u}_k^{(\mp)}\cdot\frac{d\mu}{\gamma},\ k = 1,\ 3;$$
$$\vec{S}_{2,m}(\rho,\varphi,z;\lambda) = \frac{(-i)^m}{2}\int_{-\infty}^{\infty}\omega_{\mp}^m\cdot\left(\left(\pm m\cdot\mu - \frac{\lambda^2}{\gamma} \pm \lambda^2\bar{y}_p\right)\vec{u}_1^{(\mp)} \mp \lambda^2\vec{u}_2^{(\mp)} \pm 4\mu(1-\sigma)\vec{u}_3^{(\mp)}\right)\cdot \quad (4)$$
$$\cdot\frac{e^{-i\mu\bar{x}_p \pm \gamma\bar{y}_p}d\mu}{\gamma^2},$$

где $\gamma = \sqrt{\lambda^2 + \mu^2}$, $\omega_{\mp}(\lambda,\mu) = \frac{\mu \mp \gamma}{\lambda}$, $m = 0, \pm 1, \pm 2, \ldots$;

– for transition from the layer solutions $\vec{u}_k^{(+)}$ and $\vec{u}_k^{(-)}$ to the solutions $\vec{R}_{k,m}$ of the cylindrical coordinate system

$$\vec{u}_k^{(\pm)}(x,y,z)=e^{i\mu\bar{x}_p\pm\gamma\bar{y}_p}\cdot\sum_{m=-\infty}^{\infty}(i\cdot\omega_{\mp})^m\vec{R}_{k,m},\ (k=1,\ 3);$$

$$\vec{u}_2^{(\pm)}(x,y,z)=e^{i\mu\bar{x}_p\pm\gamma\bar{y}_p}\cdot\sum_{m=-\infty}^{\infty}\left[(i\cdot\omega_{\mp})^m\cdot\lambda^{-2}\left(\left(m\cdot\mu+\bar{y}_p\cdot\lambda^2\right)\cdot\vec{R}_{1,m}\pm\gamma\cdot\vec{R}_{2,m}+4\mu(1-\sigma)\vec{R}_{3,m}\right)\right],\qquad(5)$$

where $\vec{R}_{k,m}=\tilde{\vec{b}}_{k,m}(\rho,\lambda)\cdot e^{i(m\varphi+\lambda z)}$; $\tilde{\vec{b}}_{1,n}(\rho,\lambda)=\vec{e}_\rho\cdot I_n'(\lambda\rho)+i\cdot I_n(\lambda\rho)\cdot\left(\vec{e}_\varphi\dfrac{n}{\lambda\rho}+\vec{e}_z\right)$;

$$\tilde{\vec{b}}_{2,n}(\rho,\lambda)=\vec{e}_\rho\cdot\left[(4\sigma-3)\cdot I_n'(\lambda\rho)+\lambda\rho_p I_n''(\lambda\rho)\right]+\vec{e}_\varphi i\cdot m\left(I_n'(\lambda\rho)+\frac{4(\sigma-1)}{\lambda\rho}I_n(\lambda\rho)\right)+\vec{e}_z i\lambda\rho I_n'(\lambda\rho);$$

$\tilde{\vec{b}}_{3,n}(\rho,\lambda)=-\left[\vec{e}_\rho\cdot I_n(\lambda\rho)\dfrac{n}{\lambda\rho}+\vec{e}_\varphi\cdot i\cdot I_n'(\lambda\rho)\right]$; $\vec{e}_\rho$, $\vec{e}_\varphi$, $\vec{e}_z$ – - unit vectors of the cylindrical coordinate system;

– for transition from the solutions for cylinder number p to the solutions for cylinder number q

$$\vec{S}_{k,m}(\rho_p,\varphi_p,z;\lambda)=\sum_{n=-\infty}^{\infty}\vec{b}_{k,pq}^{mn}(\rho_q)\cdot e^{i(n\varphi_q+\lambda z)},k=1,\ 2,\ 3;\qquad(6)$$

$$\vec{b}_{1,pq}^{mn}(\rho_q)=(-1)^n\tilde{K}_{m-n}(\lambda\ell_{pq})\cdot e^{i(m-n)\alpha_{pq}}\cdot\tilde{\vec{b}}_{1,n}(\rho_q,\lambda);$$

$$\vec{b}_{3,pq}^{mn}(\rho_q)=(-1)^n\tilde{K}_{m-n}(\lambda\ell_{pq})\cdot e^{i(m-n)\alpha_{pq}}\cdot\tilde{\vec{b}}_{3,n}(\rho_q,\lambda);$$

$$\vec{b}_{2,pq}^{mn}(\rho_q)=(-1)^n\left\{\tilde{K}_{m-n}(\lambda\ell_{pq})\cdot\tilde{\vec{b}}_{2,n}(\rho_q,\lambda)-\frac{\lambda}{2}\ell_{pq}\cdot\left[\tilde{K}_{m-n+1}(\lambda\ell_{pq})+\tilde{K}_{m-n-1}(\lambda\ell_{pq})\right]\cdot\tilde{\vec{b}}_{1,n}(\rho_q,\lambda)\right\}\cdot$$
$$\cdot e^{i(m-n)\alpha_{pq}},$$

where α_{pq} – angle between x_p – axis and interval ℓ_{qp}, $\tilde{K}_m(x)=(sign(x))^m\cdot K_m(|x|)$.

To meet the boundary conditions at the layer boundaries, vectors $\vec{S}_{k,m}$ in (3) using the transfer equation (4), are rewritten in the Cartesian coordinate system through the basic solutions $\vec{u}_k^{(-)}$ where $y = h$ and $\vec{u}_k^{(+)}$ where $y=-\tilde{h}$. We equate the resulting vector to the preset $\vec{U}_{\tilde{h}}^0(x,z)$ where $y=-\tilde{h}$, and for the vector obtained at $y = h$, we find the stresses, and equate them to $\vec{F}_h^0(x,z)$. We represent in advance vectors $\vec{U}_{\tilde{h}}^0(x,z)$ and $\vec{F}_h^0(x,z)$ through double Fourier integrals.

The determinant of this system of 6 equations is as follows

$$\frac{32\cdot G^3\cdot\gamma^5\cdot\mathrm{ch}\,\bar{x}\cdot\left[\bar{x}^2+(3-4\sigma)\cdot\mathrm{ch}^2\,\bar{x}+(1-2\sigma)^2\right]}{\lambda^4}$$

where $\bar{x}=\gamma(h+\tilde{h})$, G is the shear modulus. The expression in the square brackets of this determinant coincides with the known results [21].

From the obtained system of equations we find functions $H_k(\lambda,\mu)$ and $\tilde{H}_k(\lambda,\mu)$ through $B_{k,m}^{(p)}(\lambda)$.

To meet the boundary conditions for each cylinder *p*, the right-hand side of (3) applying the transfer equations (5) and (6), is rewritten in the local cylindrical coordinate system of cylinder p through basic solutions $\vec{R}_{k,m}$, $\vec{S}_{k,m}$. For the resulting vector, with $\rho_p=R_p$, we find the stresses and equate them to the preset $\vec{F}_p^0(\varphi_p, z)$ represented by the integral and the Fourier series. As a result, for each cylinder number p we obtain three infinite systems of linear algebraic equations with respect to $B_{k,m}^{(p)}(\lambda)$ which also contain $H_k(\lambda,\mu)$ and $\tilde{H}_k(\lambda,\mu)$.

The determinant of such a system relative to $B_{k,m}^{(p)}(\lambda)$ is as follows [20]:

$$\begin{aligned} &\text{where } m=0 \ \ |\Delta_0| = 8(1-\sigma)\cdot\beta^2\cdot K_1^2(\beta)\cdot K_2(\beta) \\ &\text{where } m\geq 1 \ \ |\Delta_m| > 4m\cdot K_{m-1}(\beta)K_m(x)K_{m+1}(\beta)\ ,\ \ \beta=|\lambda|\rho\ ,\ \ \lambda\neq 0. \end{aligned} \tag{7}$$

Functions $H_k(\lambda,\mu)$ and $\tilde{H}_k(\lambda,\mu)$ found earlier through $B_{k,m}^{(p)}(\lambda)$ are excluded from these equations.

As a result, we obtain for *N* cylinders a collection of 3*N* infinite systems of linear algebraic equations of second kind for determining unknown functions $B_{k,m}^{(p)}(\lambda)$.

Using (7), for the obtained systems it was proved that they are systems with a completely continuous form [5]. Hilbert's alternative and the definite solvability of the problem of the theory of elasticity allow us to conclude that the complex of these systems is uniquely solvable. Moreover, the reduction method can be applied to these systems and convergence of approximate solutions to the precise one takes place.

Functions $B_{k,m}^{(p)}(\lambda)$ found from an infinite system of equations make it possible to find expressions for $H_k(\lambda,\mu)$ and $\tilde{H}_k(\lambda,\mu)$. This will determine all unknown variables.

4. Numerical investigation of state of stress

A simulation where a truck wheel runs over a plate with two cylindrical holes lying on a rigid base is carried out. For dimensionless quantities, we introduce the coefficients of: distance *H*, load *T*=*E*·*H*/12700, where *E* is elastic modulus of the plate. The cylindrical cavities, which we denote *q* and *p*, are parallel to the layer surfaces along the horizontal axis, their radii are *Rq* = *Rp* = 7.5 / *H* (Fig. 2). The distance between the centers of the cavities is ℓ_{qp} = 25 / *H*. The upper and lower boundaries of the layer are located at the distance *h* = 15 / *H* from the center of the cylinders. The wheel width *b* = 31.5 / *H* with extension of the load application along the *z* axis in each direction by *c* = 0.2 / *H*. The wheel weighs 140·*T*. For comparison, a variant without a cylindrical cavity *p* is calculated. Poisson's ratio of the layer is $\sigma = 0.16$.

At the upper boundary of the layer, the stresses $\sigma_y^{(h)} = 0$ are preset where $|z| \geq b/2 + c$, $\dfrac{\sigma_y^{(h)}}{T} = -\dfrac{b/2 + c - |z|}{c} \cdot 10 \cdot \left((x/H)^2 + 2{,}5^2\right)^{-2}$ where $b/2 \leq |z| \leq b/2 + c$, $\dfrac{\sigma_y^{(h)}}{T} = -10 \cdot \left((x/H)^2 + 2{,}5^2\right)^{-2}$ where $|z| \leq b/2$, which is graphically shown in (Fig. 2), $\tau_{yx}^{(h)} = \tau_{yz}^{(h)} = 0$.

At the lower boundary of the layer displacements are preset $U_x^{(\tilde{h})} = U_y^{(\tilde{h})} = U_z^{(\tilde{h})} = 0$, the surfaces of the cylinders are free of stress $\sigma_\rho^{(p)} = \tau_{\rho\varphi}^{(p)} = \tau_{\rho z}^{(p)} = 0$.

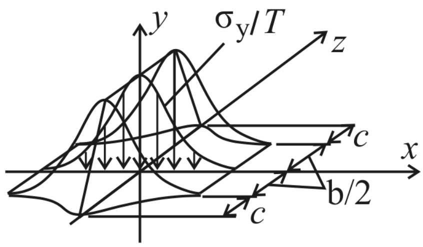
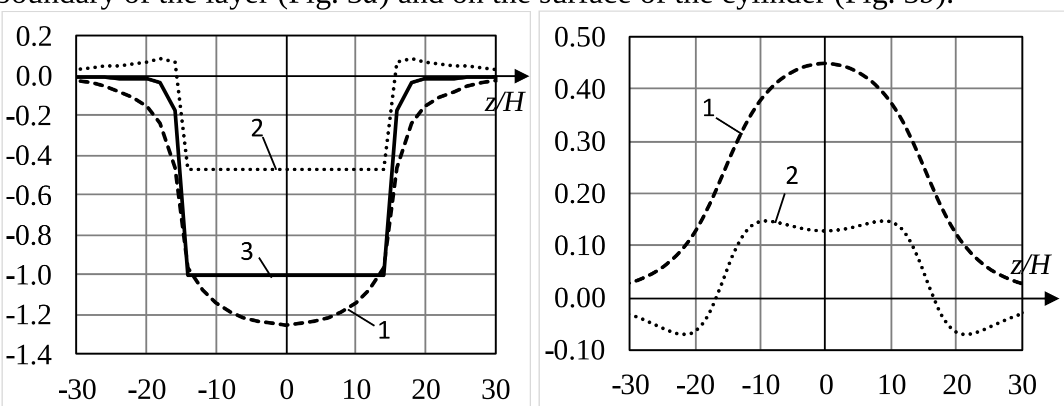


Fig. 2. Stress $\sigma_y^{(h)}/T$ at the upper boundary of layer

The infinite system was truncated by the parameter m = 10. The integrals are calculated using the quadrature formulas of Philon (for oscillating functions) and Simpson (for functions without oscillations). The precision of meeting the boundary conditions where values m are indicated and the geometric parameters are preset, is 10^{-3}.

Figure 3 shows the stresses along the z–axis where φ_q =π/2 at the upper boundary of the layer (Fig. 3a) and on the surface of the cylinder (Fig. 3b).



a *b*

Fig. 3. Normal stresses along the z-axis at φ_q =π/2: a - at the upper boundary of the layer; b - on the surface of the cavity q; 1-σ_φ/T; 2 –σ_z/T; 3 – σ_ρ/T

In Fig. 3*a*, stresses σ_ρ / T (line 3) correspond to the preset σ_y / T. Stresses σ_φ / T (Fig. 3*a*, line 1) receive maximum negative stresses where $z = 0$ and are equal to $\sigma_\varphi / T = -1.25$. The stresses σ_z (Fig. 3*a*, line 2), in addition to negative values within the wheel, are positive at the maximum value $\sigma_z / T = 0.47$.

On the surface of the cylindrical cavity along the *z*-axis (Fig. 3*b*), the stresses σ_φ / T are positive, at the maximum value $\sigma_\varphi / T = 0.45$ where $z = 0$. Stresses σ_z / T within the width of the wheel are positive, outside they are negative.

Stresses at the upper boundary of the layer along the *x*-axis are shown in Fig. 4.

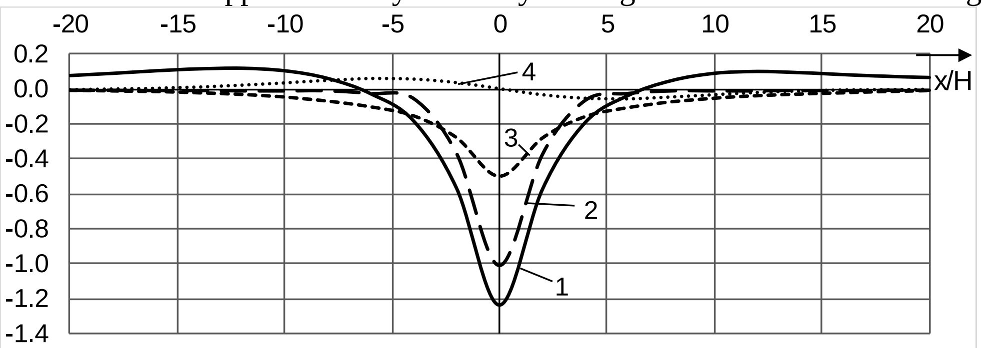
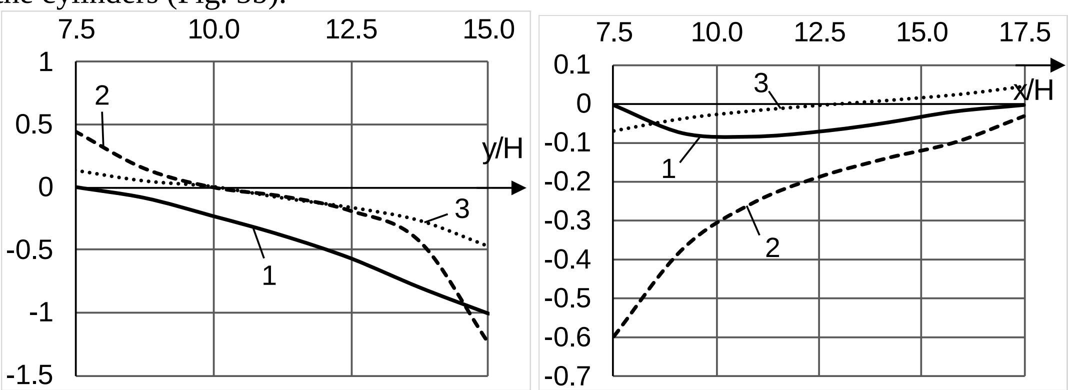


Fig. 4. Stresses at the upper boundary of the layer along the *x*-axis:
1 - σ_x / T; 2 - σ_y / T; 3 - σ_z / T; 4 - τ_{xy} / T

For the preset σ_y / T (Fig. 4, line 2), the maximum stresses are σ_x / T, which, in addition to negative (within the cavity width), also have positive values. The maximum tangent stresses τ_{xy} / T arise on the side which is free from the second cylinder (where $x/H = -4$) and are equal to $\tau_{xy} / T = 0.065$.

Fig. 5 shows the stresses on the connection between cylinder *q* and the upper boundary of the layer where $z = 0$ (Fig. 5a), as well as on the connection between the cylinders (Fig. 5b).



a *b*

Fig. 5. Normal stresses on the connections: *a* - from cylinder q to the upper boundary of the layer; *b* - from cylinder *q* to cylinder *p*; 1 - σ_ρ / T; 2 - σ_φ / T; 3 - σ_z / T

Figure 5*a* shows how stresses σ_φ / T and σ_z / T (lines 2 and 3, respectively) change sign in the interval from the upper boundary of the layer ($y / H = 15$) to the surface of the cylinder q ($y / H = 7.5$). Consequently, the upper zone of the connection is compressed and the lower one is stretched. On the connection between the cylinders (Fig. 5b), from the cylinder q to the cylinder p, stresses σ_φ / T decrease, stresses σ_z / T change sign, stresses σ_ρ / T on the surfaces of the cylinders are set to zero, but between the cylinders they grow.

Figure 6 shows how the stresses on the surface of cylinder q change, where $z = 0$, for a layer with two cavities and without cavity p.

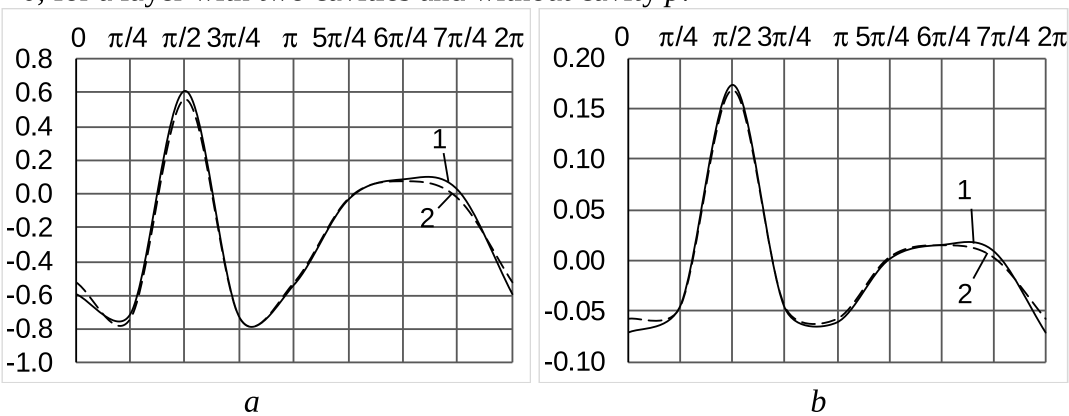


Fig. 6 Normal stresses on the surface of cavity q: *a* - σ_φ / T; *b* - σ_z / T; 1 - layer with two cavities; 2 - layer with one cavity

Maximum tensile stresses σ_φ / T and σ_z / T (Fig. 6a and Fig. 6*b*, respectively) are located in the upper part of the cavity (where $\varphi_q = \pi / 2$), the maximum compressive stresses are at $\varphi_q = \pi / 8$. Presence of cavity p (line 1) increases stresses σ_φ / T and σ_z / T on the surface of cylinder q in the upper part ($\varphi_q = \pi/2$) which is opposite the cavity p ($\varphi_q = 0$) and in the lower part ($\varphi_q = 6\pi / 4$).

5. Conclusions

A method for solving the spatial mixed problem of the theory of elasticity for a layer with several cylindrical cavities is proposed. The problem is reduced to an infinite system of linear algebraic equations. Numerical development suggests that its solution can be found with any precision by the truncation method, which is confirmed by high precision of meeting the boundary conditions.

The proposed method of solution can be used when designing structures and infrastructure whose design model includes a layer with cylindrical cavities and the considered boundary conditions.

The presented graphic charts show a picture of the stress distribution in a layer with two cylindrical cavities and a loaded upper boundary. The influence of presence of the second cavity on the stress state of the layer is analyzed. The analysis presented in the work can be used in selection of geometric parameters of structures to be designed.

It is possible to carry out further research in this direction for a layer with cylindrical cavities which is situated on a two-layer elastic base.

References

1. Bobyleva T. (2016) Approximate Method of Calculating Stresses in Layered Array. Procedia Engineering. 153: 103–106. https://doi.org/10.1016/j.proeng.2016.08.087
2. Vaysfel’d N. (2015) The axisymmetric contact interaction of an infinite elastic plate with an absolutely rigid inclusion. Acta Mech. 226: 797–810. https://doi.org/10.1007/s00707-014-1229-7
3. Popov G.Ya., Vaysfel’d N. D. (2014) Axisymmetric problem of the theory of elasticity for an infinite slab with a cylindrical inclusion, taking into account its specific weight. International Applied Mechanics. 50, № 6: 27–38.
4. Popov G.Ya. (2013) Exact solution of the mixed axisymmetric problem of elasticity theory for an infinite elastic layer weakened by cylindrical cavity. Doklady Physics. 58, Iss.8: 358–361. https://doi.org/10.1134/S1028335813080077
5. Grinchenko V.T., Ulitko A.F. (1968) An exact solution of the problem of stress distribution close to a circular hole in an elastic layer. Soviet Applied Mechanics. 10: 31–37.
6. Yingze Wang, Dong Liu, Qian Wang (2015) Effect of Fractional Order Parameter on Thermoelastic Behaviors in Infinite Elastic Medium with a Cylindrical Cavity. Acta Mechanica Solida Sinica. 28, Iss.3: 285–293. https://doi.org/10.1016/S0894-9166(15)30015-X
7. Guz' A.N., Kubenko V.D., Cherevko M.A. (1978) Diffraction of elastic waves. Nauk. Dumka, Kiev.
8. Volchkov V.V., Vukolov D.S., Storogev V.I. (2016) Diffraction of shear waves by internal tunneling cylindrical non-homogeneities in the form of a cavity and inclusion in an elastic layer with free faces. Solid mechanics. 46: 119 – 133.
9. Meleshko V.V. (2013) Equilibrium of an elastic finite cylinder under axisymmetric discontinuous normal loadings. J.Eng. Math. 78: 143 – 166. https://doi.org/10.1007/s10665-011-9524-y
10. Khoroshun L.P. (2000) Mathematical models and method of the mechanics of stochastic composites. International Applied Mechanics. 36, №10: 1284 – 1316. https://doi.org/10.1023/a:1009482032355
11. Wei Guo, Wanlin Guo (2019) Formulization of Three-Dimensional Stress and Strain Fields at Elliptical Holes in Finite Thickness Plates. Acta Mechanica Solida Sinica. 32, Iss.4: 393–430. https://doi.org/10.1007/s10338-019-00091-w
12. Yang Z. (2009): The stress and strain concentrations of an elliptical hole in an elastic plate of finite thickness subjected to tensile stress. Int J Fract.;155(1): 43–54.

13. Nikolayev A.G., Protsenko V.S. (2011) The generalized Fourier method in spatial problems of the theory of elasticity. Nats. aerokosm. universitet im. N.Ye. Zhukovskogo «KHAI», Kharkov.
14. Protsenko V.S., Nikolaev A.G. (1982) Kirsch spatial problem. Mathematical methods for analyzing dynamic systems. 6: 3 – 11.
15. Nikolayev A.G., Orlov Ye.M. (2012) Solution of the first axisymmetric thermoelastic boundary value problem for a transversely isotropic half-space with a spheroidal cavity. Problems of computational mechanics and structural durability. 20: 253–259.
16. Protsenko V., Miroshnikov V. (2018) Investigating a problem from the theory of elasticity for a half-space with cylindrical cavities for which boundary conditions of contact type are assigned. Eastern-European Journal of Enterprise Technologies. 4, № 7 (94): 43 – 50. https://doi.org/10.15587/1729-4061.2018.139567
17. Nikolayev A.G., Tanchik Ye.A. (2013) Stress distribution in a cell of a unidirectional composite material formed by four cylindrical fibers. Bulletin of the Odessa National University. Maths. Mechanics. 4: 101–111.
18. Miroshnikov V. Yu., Medvedeva A. V., Oleshkevich S. V. (2019) Determination of the Stress State of the Layer with a Cylindrical Elastic Inclusion . Materials Science Forum. 968: 413-420. https://doi.org/10.4028/www.scientific.net/MSF.968.413
19. Protsenko V. S., Ukrainets N. A. (2015) Application of the generalized Fourier method to the solution of the first main problem of the theory of elasticity in a half-space with a cylindrical cavity. Visnyk Zaporizʹkogo natsionalʹnogo universytetu. 2: 193–202.
20. Miroshnikov V.Yu. (2019) Investigation of the Stress Strain State of the Layer with a Longitudinal Cylindrical Thick-Walled Tube and the Displacements Given at the Boundaries of the Layer. Journal of Mechanical Engineering. 22, № 2: 44-52. https://doi.org/10.15407/pmach2019.02.044
21. Vorovich I.I., Aleksandrov V.M., Babeshko V.A. (1974) Non-classical mixed problems of the theory of elasticity. Nauka, Moscow.

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Received 08.11.2019