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RACK GEAR TRANSMISSION WITH ARCHED TEETH

Shevchenko S.V., Muhovaty A.A., Krol O.S.

Keywords: rack, gear, gear-cutting head, arched teeth, tooth profile, gearing equation, bending strength of teeth.

Abstract. The article presents the results of the study of rack gear transmission, whose teeth have an arched shape. This leads to the fact that the length of the teeth becomes greater than the width of the gearing, which reduces the bending stresses in the teeth, and as a consequence, increases the load capacity of the rack gear transmission.

РЕЕЧНАЯ ПЕРЕДАЧА С АРОЧНЫМИ ЗУБЬЯМИ

Шевченко С.В., Муховатый А.А., Кроль О.С.

Ключевые слова: рейка, шестерня, зуборезная головка, арочные зубья, профиль зубьев, уравнение зацепления, изгибная прочность зубьев.

Аннотация. В статье изложены результаты исследования зубчатой реечной передачи, зубья которой имеют арочную форму. Это приводит к тому, что длина зубьев становится больше ширины зубчатого зацепления, что, в свою очередь, снижает напряжения изгиба в зубьях, то есть, повышает нагрузочную способность реечной передачи.

Formulation of the problem

Develop an analytical basic for the design of the rack gear transmission with arched teeth (Fig. 1). The cutting of the arched teeth on the pinion is carried out by a standard gear-cutting head (GCH) using the single block indexing method. The rack teeth are the rounding surfaces of the pinion teeth in relative motion.

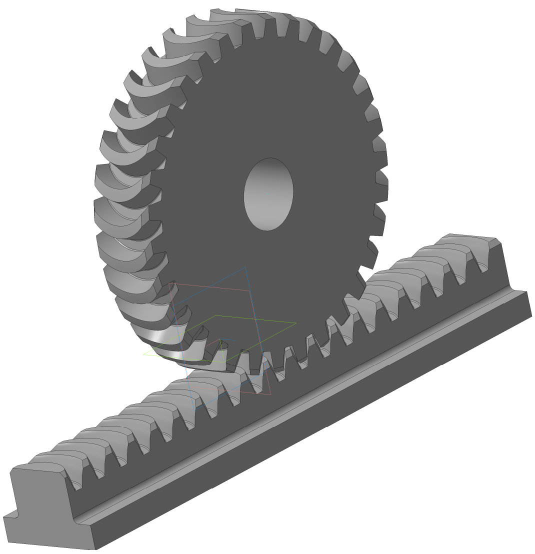

Fig. 1. Rack gear transmission scheme

Theoretical part

1) The load capacity of gear rack drives is limited mainly by the bending resistance of the teeth. One of the ways to solve this problem is constructive

modification of the rack gearing links. This work is devoted to [1-5]. Consider some analytical materials related to the transmission [5].

Pinion teeth [5] are cutting by a standard GCH using the single block indexing method, fig. 2. The cutting edges of the cutter when rotating the GCH describe in space a conical surface:

$$\begin{cases} X_И = [r_O \pm m \cdot (0{,}25 \cdot \pi - 1{,}25 \cdot tg\alpha) \pm t \cdot tg\alpha] \cdot \cos v; \\ Y_И = [-r_O \mp m \cdot (0{,}25 \cdot \pi - 1{,}25 \cdot tg\alpha) \mp t \cdot tg\alpha] \cdot \sin v; \\ Z_И = t \cdot tg\alpha. \end{cases}$$

Here r_O is the nominal radius of the GCH; $\alpha = 20^\circ$ – the angle of inclination of the cutting edges of the head cutters; m and z_1 – the face gear modulus and the number of gear teeth, respectively; t and v – independent variable parameters. The parameter t determines the position of the point on the cutting edge of the tool and varies within $[0;\ h/\cos\alpha]$, where $h = 2{,}25 \cdot m$ is the height of the cut rack gear teeth. The upper signs in these equations are for the outer edges of the cutters, the lower ones are for the inner ones.

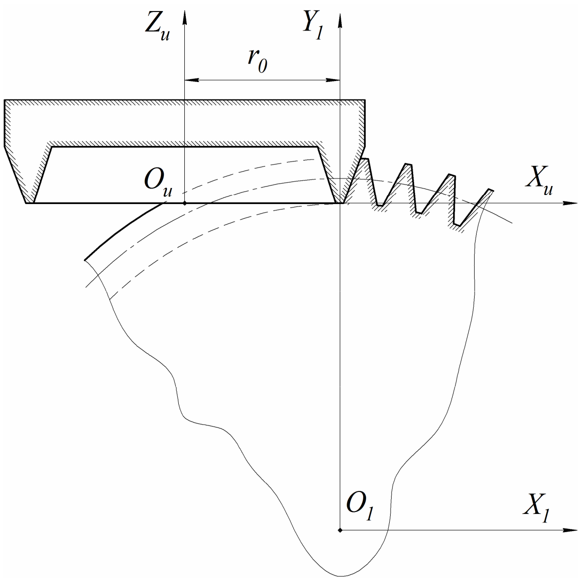

Fig. 2. Cutting teeth with single block indexing method

The transition from the coordinate system of the tool $S_И\{X_И, Y_И, Z_И\}$ to the coordinate system of the pinion $S_1\{X_1, Y_1, Z_1\}$ gives the equations of the surface of the pinion teeth:

$$\begin{cases} X_1 = r_O \cdot (1 - \cos v) \pm [m \cdot (0{,}25 \cdot \pi - 1{,}25 \cdot tg\alpha) + t \cdot tg\alpha] \cdot \cos v; \\ Y_1 = t \cdot tg\alpha \pm m \cdot (0{,}5 \cdot z_1 - 1{,}25); \\ Z_1 = [-r_O \mp [m \cdot (0{,}25 \cdot \pi - 1{,}25 \cdot tg\alpha) \mp t \cdot tg\alpha] \cdot \sin v. \end{cases} \quad (1)$$

The face profile of the pinion teeth is determined from equations (1), given by the coordinate $Z_1 = c$, where $c = [-b_W/2;\ +b_W/2]$, b_W – width of the gearing.

As a result, the equations of the face profile of the pinion teeth take the form:

$$\begin{cases} X_1 = r_O \cdot [1 - \cos v(t)] \pm [m \cdot (0{,}25 \cdot \pi - 1{,}25 \cdot tg\alpha) + t \cdot tg\alpha] \cdot \cos v(t); \\ Y_1 = t \cdot tg\alpha \pm m \cdot (0{,}5 \cdot z_1 - 1{,}25); \\ v = v(t) = \arcsin\left[\dfrac{c}{-r_O \mp m \cdot (0{,}25 \cdot \pi - 1{,}25 \cdot tg\alpha) \mp t \cdot tg\alpha]}\right]. \end{cases} \quad (2)$$

To compose the equations of the teeth rack face profile, we use the kinematic method for obtaining the gearing equation $\vec{n}_1 \cdot \vec{V}_1^{(12)} = 0$ [6]. The normal vector to the pinion teeth profile (2):

$$\vec{n}_1 = -tg\alpha \cdot \vec{i}_1 + \{\dot{v}_t \cdot \sin v \cdot [r_O \mp m \cdot (0{,}25 \cdot \pi - 1{,}25 \cdot tg\alpha) \mp t \cdot tg\alpha] \pm tg\alpha \cdot \cos v\} \cdot \vec{j}_1 .$$

Relative velocity vector:

$$\vec{V}_1^{(12)} = \omega_1 \cdot [Y_1 - r_1) \cdot \vec{i}_1 + (r_1 \cdot \varphi_1 - X_1) \cdot \vec{j}_1] .$$

As a result, the gearing equation $\vec{n}_1 \cdot \vec{V}_1^{(12)} = 0$ takes the form:

$$Y_1 \cdot tg\alpha - (r_1 \cdot \varphi_1 - X_1) \cdot \{\dot{v}_t \cdot [r \mp m \cdot (0{,}25 \cdot \pi - 1{,}25 \cdot tg\alpha) \mp t \cdot tg\alpha] \cdot \sin v(t) \pm \\ \pm tg\alpha \cdot \cos v(t)\} = 0. \quad (3)$$

The transition from the coordinate system of the rotating pinion $S_1\{X_1,\ Y_1\}$ to the coordinate system $S_2\{X_2, Y_2\}$ associated with the translational moving rack with speed $\vec{V}_2 = \vec{\omega}_1 \times r_1$ by using expressions:

$$\begin{cases} X_2 = r_O \cdot [1 - \cos v(t)] \pm [m \cdot (0{,}25 \cdot \pi - 1{,}25 \cdot tg\alpha) + t \cdot tg\alpha] \cdot \cos v(t) - r_1 \cdot \varphi_1; \\ Y_2 = t \cdot tg\alpha \pm m \cdot (0{,}5 \cdot z_1 - 1{,}25) - r_1, \end{cases} \quad (4)$$

where $r_1 = 0{,}5 \cdot m \cdot z_1$ – the radius of the pitch circle pinion (fig. 3).

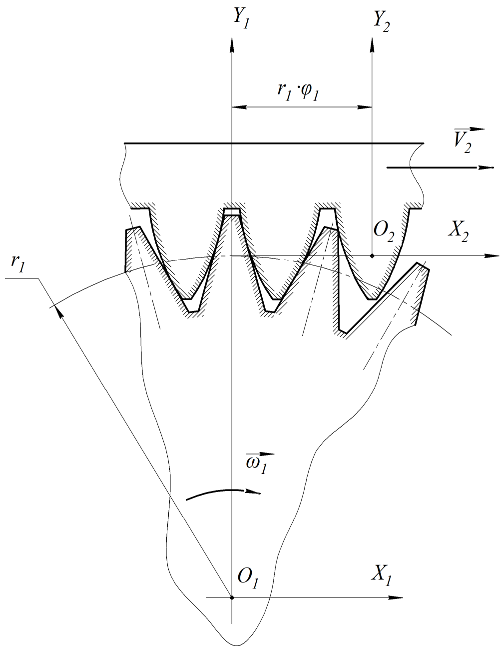

Fig. 3. Pitch circle radius of the pinion

The face profile of the rack teeth is determined by equations (4) together with the third equation of the system (2) and the gearing equation (3).

2) With certain combinations of pinion and GCH sizes, secondary cutting is possible. To eliminate this phenomenon, it is necessary to perform the condition: $r_O > r_{O(MIN)}$, where $r_{O(MIN)}$ – minimum possible radius of the GCH, under which secondary cutting is absent:

$$r_{O(MIN)} \approx m \cdot (\sqrt{2{,}25 \cdot z_1} + 0{,}33).$$

Experimental part

For example, for transmission with parameters $m = 5$ mm and $z_1 = 27$:

$$r_{O(MIN)} \approx 5 \cdot (\sqrt{2{,}25 \cdot 27} + 0{,}33) \approx 40{,}6\,mm \text{ or } d_{O(MIN)} \approx 81\,mm$$

That is, you should choose according to GOST 11902-77 a GCH with a nominal diameter $d_O = 100\,mm$ or more.

In addition to the condition $r_O > r_{O(MIN)}$, a quite obvious relation must be fulfilled connecting the radius of the GCH r_O with the width of the gearing b_W: $r_O > 0{,}5 \cdot b_W$. So, for the above transmission parameters $b_W = 120\,mm$, we get:

$$r_O > 0{,}5 \cdot 120 = 60\,\text{mm}, \text{ or } d_O > 120\,mm,$$

that is, you should adopt a standard head with $d_O = 125\,\text{m}m$ and more.

Therefore, of the two restrictions on the parameter: r_O ($r_O > r_{O(MIN)}$) and, $r_O > 0{,}5 \cdot b_W$, one should take the one that gives the greater value r_O.

3) The arched shape of the teeth in the longitudinal direction leads to an increase in their length compared to the spur gear [7, 8] with the same value b_W. The length of the arched teeth: $L_A = d_O \cdot \arcsin(b_W / d_O)$. Since the spur gearing [7, 8], the length of the teeth L is equal to the width of the gearing b_W, that is $L = b_W$, the increment in the length of the arched shape teeth will be:

$$K_L = L_A / L = \frac{d_O}{b_W} \cdot \arcsin(b_W / d_O).$$

In the first approximation, this means that the bending stress of the rack arched teeth will be in K_L-less than that of straight teeth. For transmission with the above gearing parameters with $d_O = 160\,мм$:

$$K_L = \frac{160}{120} \cdot \arcsin(120/160) \approx 1{,}13,$$

that is, the reduction of bending stresses in the arched teeth as compared with straight teeth will be at least 13%. The actual margin of arched teeth bending strength is expected even more due to the increase in their thickness as the distance from the average face section of the pinion.

Conclusion

Primary analytical dependencies for complex geometro-kinematic analysis of rack gear transmission with arch-shaped teeth are obtained. The relationship between the transmission parameters and the limiting size of the gear cutting head has been established. An increase in the load capacity of the arch teeth in comparison with the straight teeth of the rack gear transmission, subject to all other conditions being equal is proved.

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Шевченко Святослав Владимирович – к.т.н., доцент, заведующий кафедрой машиноведения, Луганский национальный университет им. В.Даля, г.Луганск, Украина	**Shevchenko Svyatoslav Vladimirovich –** Ph.D., Associate Professor, head of Department of mechanical engineering, Volodymyr Dahl Lugansk National University, Lugansk, Ukraine
Муховатый Александр Анатольевич – к.т.н., доцент, доцент кафедры машиноведения, Луганский национальный университет им. В.Даля, г.Луганск, Украина	**Muhovaty Alexander Anatol'evich** – Ph.D., Associate Professor of Department of mechanical engineering, Volodymyr Dahl Lugansk National University, Lugansk, Ukraine
Кроль Олег Семенович – к.т.н., доцент, профессор кафедры машиностроения, станков и инструментов, Восточноукраинский национальный университет им. В.Даля, г.Северодонецк, Украина, krolos@yandex.ru	**Krol Oleg Semenovich –** Ph.D., Associate Professor of Department of mechanical engineering, machine tools and instruments, Volodymyr Dahl East-Ukrainian National University, Severodonetsk, Ukraine, krolos@yandex.ru

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