УДК 62.752, 621:534.833;888.6 https://doi.org/10.26160/2309-8864-2019-7-42-50

ДИНАМИЧЕСКИЕ ВЗАИМОДЕЙСТВИЯ МЕЖДУ ЭЛЕМЕНТАМИ МЕХАНИЧЕСКИХ КОЛЕБАТЕЛЬНЫХ СИСТЕМ. ВОЗМОЖНОСТИ ОЦЕНКИ СИЛОВЫХ ПАРАМЕТРОВ

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Ключевые слова: механические колебательные система, структурные схемы, реакции связей, параметры динамического состояния.
Аннотация. Рассматриваются вопросы использования реакций связей как критерия оценки динамического состояния механических колебательных система. Предлагаемый подход основан возможностях преобразования структурных схем эквивалентных в динамическом отношении систем автоматического управления. Трансформация таких расчетных схем позволяет выделить в механической колебательной систем объект защиты и отрицательную обратную связь, являющуюся динамической реакцией. Приведен ряд примеров амплитудно-частотных характеристик, полученных при различных значениях жесткости одного из упругих элементов.

DYNAMIC INTERACTIONS BETWEEN ELEMENTS OF MECHANICAL OSCILLATION SYSTEMS. POSSIBILITIES ESTIMATION OF FORCE PARAMETERS

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Keywords: mechanical oscillatory system, structural diagrams, constraint reactions, dynamic state parameters.
Abstract. The use of responses of constraints as a criterion for estimating the dynamic state of mechanical oscillatory systems is considered in the article. The proposed approach is based on the possibilities of transforming the structural diagrams of the dynamically equivalent automatic control systems. The transformation of such computational schemes makes it possible to isolate in the mechanical oscillatory system the object of protection and negative feedback, which is a dynamic reaction. A number of examples of amplitude-frequency characteristics obtained for different values of the stiffness of one of the elastic elements are given in the paper.

Introduction

In solving the problems of the dynamics of machines and equipment under the influence of vibratory external influences, in particular, in problems of vibration protection, the coordinates of objects whose frequency dependences are revealed in the frequency characteristics of the system are usually used as parameters of the dynamic state [1÷3]. The transfer functions of mechanical oscillatory systems reflect the basic properties of systems associated with the consideration of dynamic effects such as resonances, dynamic damping of oscillations, et al.

At the same time, the notion of dynamic reactions of constraints arising at the points of connection of the elementary links of the system with each other, as well as at points of contact with supporting surfaces and the object of protection, are of great importance: Some features of the determination of dynamic reactions in mechanical oscillatory systems were reflected in [4, 5].

In the problems of vibration protection of technical objects within the scope of the structural theory of oscillatory systems, the constraint reaction is interpreted as an inverse negative relationship in the structural diagram of the dynamically equivalent automatic control system. In this case, the protection object is interpreted as an integrating link of the second order, and physically, the inverse negative connection corresponds to the representation of the unit dynamic stiffness of the generalized spring [6].

To a lesser extent, the properties of the reactions of constraints arising at points of contact or connections of elements of a mechanical system with each other are studied. Of particular importance, in this respect, is the estimation of the values of dynamic reactions at characteristic points of the system that determine the reliability and safety of the system as a whole. In this sense, the value of constraint reactions can be considered as parameters of the dynamic state, as well as coordinates, velocities and accelerations of the movement of the protection object.

In the present article, the features of the formation of constraint reactions in linear mechanical oscillatory systems with two degrees of freedom under the action of harmonic external perturbations in the concepts of use of reactions as parameters of the dynamic state of the system are considered.

I. General provisions. Peculiarities of the research problem formulation

The generalized computational scheme for solving the problems of the dynamics of objects in systems with two degrees of freedom is presented in Figure 1. In addition to the concentrated masses m_1 and m_2, the system contains elastic elements with stiffness coefficients k_1, k_2, k_3. The system has two support surfaces I and II, which can perform harmonic motions z_1, z_2. The motion of the system is considered in the fixed basis using the coordinates y_1 and y_2. At the contact points (pp. $A \div B$), restraining (or bilateral) constraints are assumed [7]. The equation of motion of the system under the kinematic perturbation (Q_1=0, Q_2=0) has the form of

$$m_1\ddot{y}_1 + y_1(k_1 + k_2) - k_2 y_2 = k_1 z_1(t) + Q_1(t), \quad (1)$$

$$m_2\ddot{y}_2 + y_2(k_2 + k_3) - k_2 y_1 = k_3 z_2(t) + Q_2(t). \quad (2)$$

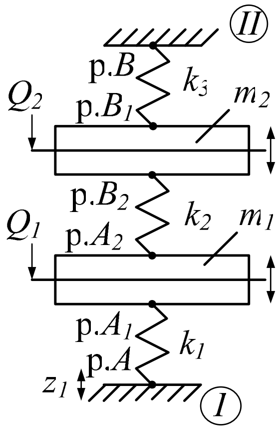

Fig. 1. The generalized computational scheme of a mechanical oscillatory system with two degrees of freedom: *I*, *II* are supporting surfaces; pp. $A \div B$ are points of contacts; Q_1, Q_2 are power perturbations; z_1, z_2 are the kinematic perturbations

The corresponding structural diagrams in various forms of a detailed representation of the possibilities of their transformations are shown in Fig. 2 a÷e.

Construction of the structural diagrams requires a Laplace transformation [1, 2] with subsequent construction of structural diagrams [4÷6].

Using Laplace transforms, one can reduce (1), (2) to the forms:

$$m_1 p^2 \bar{y}_1 + \bar{y}_1(k_1 + k_2) - k_2 \bar{y}_2 = k_1 \bar{z}_1 + \overline{Q}_1(t), \tag{3}$$

$$m_2 p^2 \bar{y}_2 + \bar{y}_2(k_2 + k_3) - k_2 \bar{y}_1 = k_3 \bar{z}_2 + \overline{Q}_2(t), \tag{4}$$

where p = jω is a complex variable; , $\bar{y}, \bar{z}, \overline{Q}$, are the images of *y(t), z(t)* и *Q(t)* in the region of Laplace transforms.

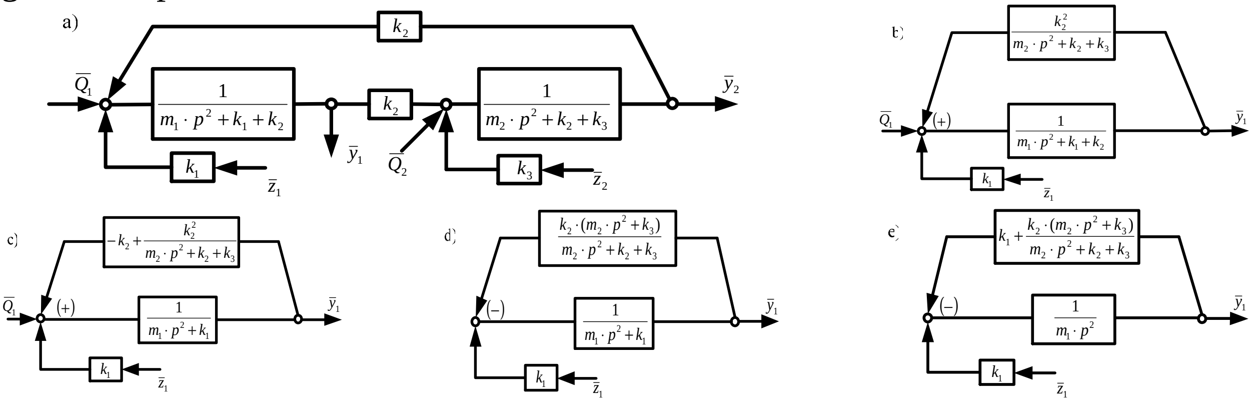

Fig. 2. Variants of representation of the structural diagram: a is a structural diagram of a general form, corresponding to the mathematical model; b is a transformed structural diagram with the exception of the coordinate $\bar{y}_2$ (for $z_2 = 0$, $Q_2=0$); c is a transformed structural diagram with the exception of the coordinate and the removal of k_2 in the feedback (at $z_2=0$, $Q_2=0$); d is a transformed structural diagram with the formation of a transfer function in the form of a fractional-rational expression in the scope of the partial system $m_1p^2+k_1$ (for $z_2=0$, $Q_2=0$, $Q_1=0$); e is a transformed structural diagram with allocation of the protection object m_1 as an integrating link of the second order (for $z_2=0$, $Q_2=0$, $Q_1=0$)

Fig. 2 shows the possibilities of transformation of structural diagrams with the identification of the necessary elements or blocks of the general structural diagram (Figure 2a). The determination of transfer functions for various types and combinations of external influences is possible on the basis of the superposition principle [1, 2]. Conventional methods of construction are related as rules with the selection of a situation where one input and one output are defined in the system. For linear systems with several input effects of the same frequency, transformations with the construction of equivalent input influences are possible.

The research objective is to develop a method for constructing constraint reactions at the characteristic points of a mechanical oscillatory system and to evaluate the possibilities of using coupling reactions as parameters of the dynamic state of the protection object.

II. Peculiar features of transformation of computational schemes and structural diagrams of the vibration system

Let's consider variants of transformation of computational schemes, choosing various relationships of system parameters, possible changes in the position of support surfaces as shown in Fig. 3a–h.

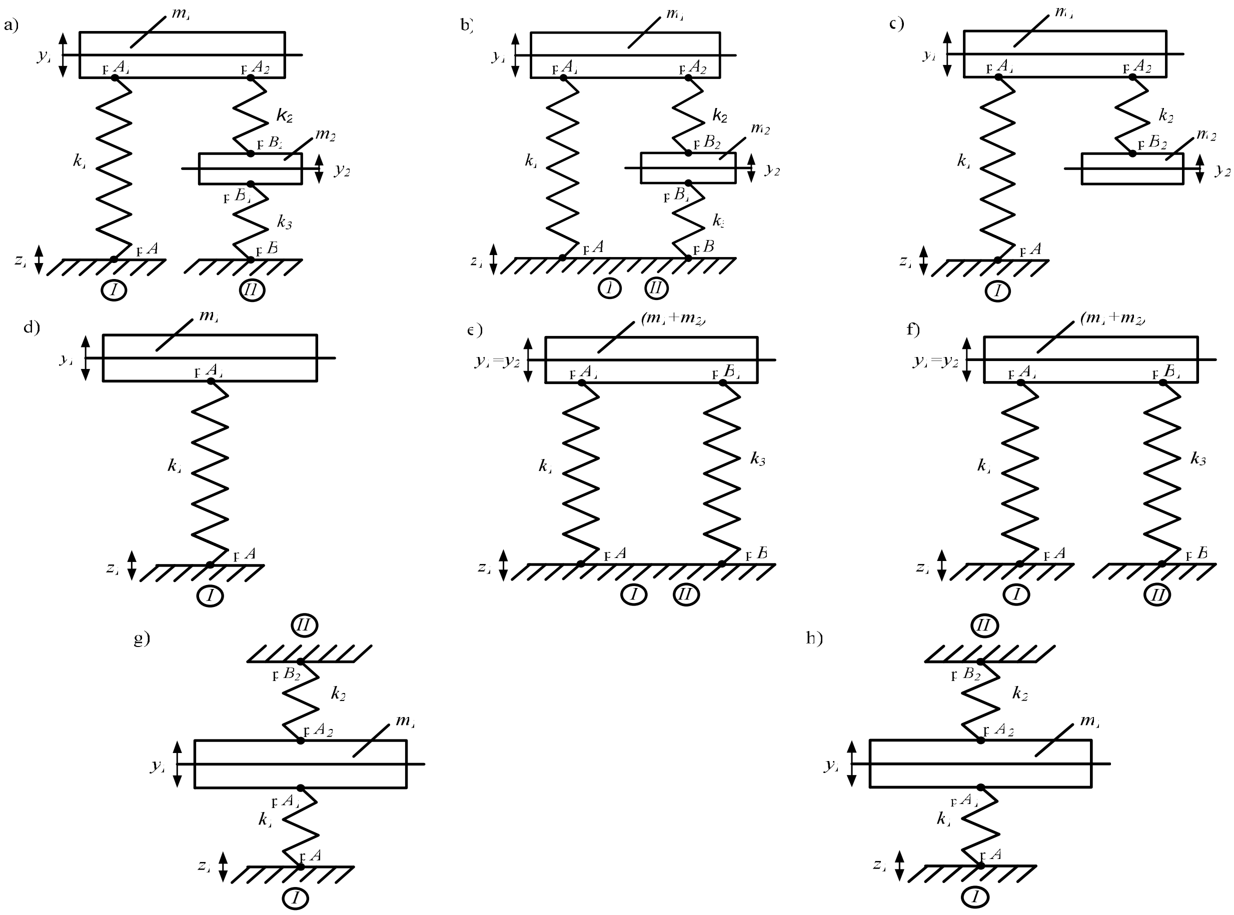

Fig. 3. Variants of calculation schemes: a is a computational scheme with divided support surfaces; b is a computational scheme with the combined support surfaces; c – there are no points B and B_1 (k_3=0*);* in the computational scheme; d is a computational scheme with divided support surfaces for $k_2 = 0$ *u* $k_3 = 0$; e is a computational scheme with combined support surfaces for $k_2 \to \infty (y_1 = y_2)$*;* f is a computational scheme with divided support surfaces for $k_2 \to \infty (y_1 = y_2)$*;* g is a computational scheme with divided support surfaces for $k_3 \to \infty$; h is a computational scheme with divided support surfaces for $m_2 \to \infty$ and $k_3 \neq 0$ or $k_3 = 0$

The calculation scheme in Fig. 3a corresponds to the representation of the support surface consisting of two parts, which implies the action of various kinematic perturbations $z_1(t)$, $z_2(t)$ on the object of protection m_1. From the side of the support surface *I*, the constraint reaction is formed by an elastic element with a stiffness k_1; from the side of the surface *II*, a constraint reaction takes place on the protection object m_1, which at the point A_2 is formed by a mechanical chain of consecutively connected elements to k_3, m_2 and k_2. In Fig.3b, the support surfaces *I* and *II* are combined, which corresponds to supporting the protection object by one "combined vibration isolator", which determines the overall reaction of the constraints to one support surface. For k_3=0, as shown in Fig. 3c, in the vibration protection system at $z_1 \neq 0$, (Q_1=0, Q_2=0) the dynamic damping mode can appear at the protection object m_1 in the kinematic form of the external perturbation.

For k_2=0 and k_3=0, the initial computational scheme (Fig. 3a) is transformed into a system with one degree of freedom (Fig. 3d). For $k_2 \to \infty$ the form of motion of two masses m_1 and m_2 with the tendency $y_1 \to y_2$ (or $y_1 = y_2$) is formed, and the system as a whole is defined in the low-frequency range by the parameters of the motion of the system with one degree of freedom (m_1+m_2), (k_1+k_3).

In Fig. 3f it is assumed that the motion for m_1+m_2 can occur with two perturbing factors z_1 and z_2 (Q_1=0, Q_2=0). In case when $k_3\to\infty$, the element m_2 becomes a part of the support surface *II* (Figure 3f) and the system acquires one degree of freedom.

A similar result can be obtained for $k_3\neq0$ and $m_2\to\infty$ (Fig. 3h). If k_3=0, then in this case, for $m_2\to\infty$, the system will also have one degree of freedom in the vibrational form of the motion.

III. Determining transfer functions

We use the calculation scheme in Fig. 1 and the structural diagram in Fig. 2a. In this case, the transfer functions are defined:

$$W_1(p)=\frac{\bar{y}_1}{\bar{z}_1}=\frac{k_1(m_2p^2+k_2+k_3)}{A_0}, \quad (5) \qquad W_2(p)=\frac{\bar{y}_2}{\bar{z}_1}=\frac{k_1k_2}{A_0}, \quad (6)$$

where A_0 is a frequency characteristic equation

$$A_0=(m_1p^2+k_1+k_2)(m_2p^2+k_2+k_3)-k_2^2. \quad (7)$$

The kinematic perturbation in this case (the system is linear) is replaced by an equivalent force action

$$\overline{Q}_{eq.1}=k_1\bar{z}_1. \quad (8)$$

then

$$W_3(p)=\frac{\bar{y}_1}{\overline{Q}_{eq1}}=\frac{m_2p^2+k_2+k_3}{A_0}, \quad (9) \quad W_4(p)=\frac{\bar{y}_2}{\overline{\mathrm{Q}}_{eq.1}}=\frac{k_2}{A_0}. \quad (10)$$

It is obvious that $W_1(p)=k_1W_3(p)$ and $W_2(p)=k_1W_4(p)$.

IV. Determining dynamic reactions of constraints at characteristic points (pp. *A*÷*B*)

The calculation scheme in Fig. 1a or Fig. 3a is considered; these schemes are equivalent. All the relations are written in the operator form ($\bar{y}_1, \bar{y}_2$).

The reaction at point *A* is determined by the formula:

$$\overline{R}_A=k_1\bar{y}_1=\frac{k_1^2\bar{z}_1\left(m_2p^2+k_2+k_3\right)}{A_0}, \quad (11)$$

or

$$\overline{R}_A=\frac{k_1\left(m_2p^2+k_2+k_3\right)}{A_0}\overline{Q}_{eq.1}. \quad (12)$$

Let us introduce the concepts of the transfer function by the constraint reaction. The input signal is a kinematic perturbation $\bar{z}_1$ or an equivalent force action $\overline{Q}_{eq.1}=k_1\bar{z}_1$. We obtain two transfer functions:

$$W'_{\overline{\mathrm{R}}_\mathrm{A}}(p)=\frac{\overline{R}_A}{\bar{z}_1}=\frac{k_1^2(m_2p^2+k_2+k_3)}{A_0}, \quad (13) \quad W''_{\overline{\mathrm{R}}_\mathrm{A}}(p)=\frac{\overline{R}_A}{\overline{Q}_{eq.1}}=\frac{k_1(m_2p^2+k_2+k_3)}{A_0}. \quad (14)$$

From (14), in particular, it follows that the ratio of the amplitude of the oscillations of the dynamic reaction to the equivalent force $\overline{Q}_{eq.1}$ generated by the kinematic perturbation has a form of the transfer function of the system when the input signal is considered to be a kinematic perturbation $\bar{z}_1$, and displacement with respect to the coordinate $\bar{y}_1$ is respectively considered to be the output signal. This is easily verified by structural diagrams in Fig.2.

The reaction at point A_1 is formed as a result of deformation of a linear elastic element (spring) with stiffness k_1. In this case, we can assume that the statement "actio est reactio" is valid,

$$\bar{R}_A = -\bar{R}_{A_1}, \tag{15}$$

that is, the reactions are equal in magnitude and are directed to the opposite sides (in accordance with Newton's third law). However, if a mass-and-inertia element is placed between the points A and A_1, or the mass-and-inertia properties of the spring itself are taken into account, then such a condition is no longer observed. Let us turn to the calculation scheme in Fig. 3a. It should be noted that the mass-and-inertial element m_1 is supported as follows.

One leg of the support is an elastic element k_1 with fixing points A and A_1, respectively, to the support surface *I* and mass m_1. The second branch is formed from the successive interconnections of elastic elements k_2, a mass-and-inertia element m_2 and an elastic element with stiffness k_3. Accordingly, the branch has contact points at point B with the support surface *II*, and at point A_2 with mass m_1. In this case:

$$\bar{R}_{A2} \neq \bar{R}_B. \tag{16}$$

The reaction at point B can be found

$$\bar{R}_B = k_3 \bar{y}_2 = \frac{k_1 k_2 k_3 \bar{z}_1}{A_0}. \tag{17}$$

We get two transfer functions:

$$W'_{\bar{R}_B}(p) = \frac{\bar{R}_B}{\bar{Z}_1} = \frac{k_1 k_2 k_3}{A_0}, \tag{18}$$

$$W''_{\mathrm{R_B}}(p) = \frac{\bar{R}_B}{\bar{Q}_{eq.1}} = \frac{k_2 k_3}{A_0}. \tag{19}$$

The reaction at point B_1, where the spring k_3 is in contact with the mass-and-inertia element m_2, will be determined:

$$\bar{R}_{B_1} = -\bar{R}_B. \tag{20}$$

To find the reaction at point A_2, it is necessary to find the unit stiffness at point A_2. It is at this point that the second branch (k_3, k_2, m_2) has contact with the element m_1.

The unit stiffness will be:

$$k_{A_2 US} = \frac{k_2 (m_2 p^2 + k_3)}{m_2 p^2 + k_2 + k_3}. \tag{21}$$

The unit stiffness reflects the dynamic stiffness of the successive chain consisting of k_3, $m_2 p^2$ and k_2. The features of this approach are presented in monographs [1, 2]. A similar result can be obtained using the structural transformations of the original structural diagram in Fig. 2a. In this case, on the transformed structural diagram in Fig. 2c in the negative feedback chain with respect to the protection object with the transfer function $\frac{1}{m_1 p^2 + k_1}$, has a corresponding transfer function.

$$W_{fb}(p) = k_{A_2US} = \frac{k_2(m_2 p^2 + k_3)}{m_2 p^2 + k_2 + k_3}. \tag{22}$$

The expression obtained in this way completely coincides with the expression (19) for unit stiffness.

Knowing the unit stiffness k_{A2US} , we find the dynamic response at point A_2

$$\overline{R}_{A_2} = k_{A_2US}\overline{y}_1 = \frac{k_1 k_2(m_2 p^2 + k_3)\overline{z}_1}{A_0}. \tag{23}$$

We introduce the transfer function for the dynamic reaction at point A_2

$$W'_{\overline{R}_{A_2}}(p) = \frac{\overline{R}_{A_2}}{\overline{z}_1} = \frac{k_1 k_2(m_2 p^2 + k_3)}{A_0}, \ (24) \quad W''_{\overline{R}_{A_2}}(p) = \frac{\overline{R}_{A_2}}{\overline{Q}_{eq.1}} = \frac{k_2(m_2 p^2 + k_3)}{A_0}. \ (25)$$

The dynamic reaction at the protection object, that is, in the mass-and-inertia element m_1, is determined by summing the reactions at points A_1 and A_2.

Thus,

$$\overline{R}_{m_1}(p) = \overline{R}_{A_1} + \overline{R}_{A_2} = \frac{k_1\overline{z}_1[k_1(m_2 p^2 + k_2 + k_3) + k_2(m_2 p^2 + k_3)]}{A_0}$$

or

$$\overline{R}_{m_1}(p) = \frac{k_1(k_1 + k_2)m_2 p^2 + k_1^2(k_2 + k_3) + k_1 k_2 k_3}{A_0}\overline{z}_1. \tag{26}$$

Accordingly, we obtain the transfer functions for the mass-and-inertia element m_1

$$W'_{\overline{R}_{m_1}}(p) = \frac{\overline{R}_{m_1}(p)}{\overline{z}_1} = \frac{k_1(k_1 + k_2)m_2 p^2 + k_1^2(k_2 + k_3) + k_1 k_2 k_3}{A_0}, \tag{27}$$

$$W''_{\overline{R}_{m_1}}(p) = \frac{\overline{R}_{m_1}(p)}{\overline{Q}_{eq.1}} = \frac{(k_1 + k_2)m_2 p^2 + k_1(k_2 + k_3) + k_2 k_3}{A_0}. \tag{28}$$

Similarly, we can find a dynamic reaction that appears on an element of mass m_2.

In this case, we can imagine that the mass m_2 reflects two elastic branches at points B_1 and B_2. The dynamic reaction at the point B_1 is determined by the expression (21). In turn, the second branch is formed by a mechanical chain of the consecutively connected elements k_1, m_1p^2 and k_2, which allows, in accordance with the rules of transformation of mechanical circuits, finding the unit stiffness at the point B_2

$$k_{B_2US} = \frac{k_2(m_1 p^2 + k_1)}{m_1 p^2 + k_1 + k_2}. \tag{29}$$

From (29) we determine the dynamic reaction at the point B_2

$$\overline{R}_{B_2} = k_{B_2US} \cdot \overline{y}_2 = \frac{k_1 k_2^2(m_1 p^2 + k_1)\overline{z}_1}{(m_1 p^2 + k_1 + k_2)A_0}. \tag{30}$$

The transfer functions of dynamic constraints at the point B_2 take the form:

$$W'_{\overline{R}_{B_2}} = \frac{\overline{R}_{B_2}}{\overline{z}_1} = \frac{k_1 k_2^2(m_1 p^2 + k_1)}{(m_1 p^2 + k_1 + k_2)A_0}, \ (31) \quad W''_{\overline{R}_{B_2}} = \frac{\overline{R}_{B_2}}{\overline{Q}_{eq.1}} = \frac{k_2^2(m_1 p^2 + k_1)}{(m_1 p^2 + k_1 + k_2)A_0}. \ (32)$$

The total dynamic reaction formed on the mass-and-inertia element m_2 is determined as the sum of two dynamic reactions $\overline{R}_{B1}$ and $\overline{R}_{B1}$. Thus, we finally write down that

$$\overline{R}_{m_2}(p) = \overline{R}_{B_1} + \overline{R}_{B_2} = \frac{k_1 k_2 \bar{z}_1[(k_2+k_3)m_1 p^2 + k_3(k_1+k_2) + k_1 \cdot k_2]}{(m_1 p^2 + k_1 +}. \quad (33)$$

The transfer functions of the dynamic constraints for the mass-and-inertia element m_2 will take the form:

$$W'_{\overline{R}_{m_2}} = \frac{\overline{R}_{m_2}}{\bar{z}_1} = \frac{k_1 k_2[(k_2+k_3)m_1 p^2 + k_3(k_1+k_2) + k_1 k_2]}{(m_1 p^2 + k_1 + k_2)A_0}. \quad (34)$$

$$W''_{\overline{R}_{m_2}} = \frac{\overline{R}_{m_2}}{\overline{Q}_{eq.1}} = \frac{k_2[(k_2+k_3)\cdot m_1 p^2 + k_3(k_1+k_2) + k_1 k_2]}{(m_1 p^2 + k_1 + k_2)\cdot A_0}. \quad (35)$$

Preliminary analysis (35) allows establishing the properties of the amplitude-frequency characteristic (AFC), which are determined by the fact that:

1. The dynamic reaction $\overline{R}_{m2}$ takes extreme values three times (twice at frequencies of natural oscillations or at resonances, and also at the partial frequency $\omega^2 = \frac{k_1 + k_2}{m_1}$);

2. At the oscillation frequency determined from the frequency equation of the numerator (35), a mode of the dynamic mode is formed, which could be called the "zeroing of the dynamic reaction"; in addition, the frequency is defined by the expression

$$\omega^2 = \frac{k_1 \cdot k_2 + k_3 \cdot (k_1 + k_2)}{m_1 \cdot (k_2 + k_3)}. \quad (36)$$

Conclusion

1. Dynamic constraint reactions can serve as parameters of the state of a mechanical oscillatory system as well as known forms of estimation based on the use of kinematic parameters.

2. The dynamic reaction of the constraints at the selected point of the system is defined in the operator form as the product of the displacement by the unit dynamic stiffness and carries information about the features of the resonance modes and the dynamic damping of the oscillations.

3. Methods of structural transformations are proposed for obtaining dynamic reactions, which are based on the use of the parameters of the feedback chains formed with respect to the selected mass-and-inertia elements.

4. The effect of the maximum of the constraint reaction is discovered, which is physically treated as an increase in the unit dynamic stiffness at the frequency corresponding to the mode of dynamic damping of oscillations.

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